PHOTOELECTRON SPECTROSCOPY OF DIATOMIC MOLECULES: OPTIMIZED GREEN’S FUNCTIONS AND DENSITY FUNCTIONAL APPROACH

We present the optimized version of the hybrid combined density functional theory (DFT) and the Green’s-functions (GF) approach to quantitative treating the diatomic photoelectron spectra. The Fermi-liquid quasiparticle version of the density functional theory is used. The density of states, which describe the vibrational structure in photoelectron spectra, is defined with the use of combined DFT-GF approach and is well approximated by using only the first order coupling constants in the optimized one-quasiparticle approximation. Using the combined DFT-GF approach leads to significant simplification of the calculation and increasing an accuracy of theoretical prediction.

Introduction

In this paper we present the optimized version of the hybrid combined density functional theory (DFT) and the Green’s-functions (GF) approach to quantitative treating the diatomic photoelectron spectra.

The approach is based on the Green’s function method (Cederbaum-Domske version) [11,12], Fermi-liquid DFT formalism [1-8] and use of the novel effective density functionals (see also [13-16]). The density of states is well approximated by using only the first order coupling constants in the one-particle approximation. It is important that the calculational procedure is significantly simplified with using the quasiparticle DFT formalism. Thus quite simple method becomes a powerful tool in interpreting the vibrational structure of photoelectron spectra for different molecular systems.

As usually (see details in refs. [1-12]), the quantity which contains the information about the ionization potentials (I.P.) and molecular vibrational structure due to quick ionization is the density of occupied states:

$$N_k(\epsilon) = \langle \Psi_0 | \sum_i \epsilon_i a_i^\dagger a_i | \Psi_0 \rangle$$

where $|\Psi_0\rangle$ is the exact ground state wave function of the reference molecule and is an electron destruction operator, both in the Heisenberg picture. Usually in order to calculate the value (1) states for photon absorption one should express the Hamiltonian of the molecule in the second quantization formalism.

2. Theory: Density of states in one-body and many-body solution

As usually, introducing a field operator

$$\sum a_i$$

with the Hartree-Fock (HF) one-particle functions $\phi_i(\epsilon)$ are the one-particle HF energies and $\epsilon$ denotes the set of orbitals occupied in the HF ground state; $R_0$ is the equilibrium geometry on the HF level) and dimensionless normal coordinates $Q_s$ one can write the standard Hamiltonian as follows [2,11]:

$$H = H_E + H_N + H_V^{(1)} + H_V^{(2)} =$$

$$H_E = \sum_i c_i(R_0) a_i^\dagger a_i + \frac{1}{2} \sum_{i,j,k} V_{\alpha\beta}(R_0) a_i^\dagger a_j^\dagger a_k a_j -$$

$$- \sum_{i,f} \sum_{k,j} [W_{\alpha\beta}(R_0) - V_{\alpha\beta}(R_0)] a_i^\dagger a_j$$

$$H_N = \hbar \sum_{i=1}^N \omega_i (b_i^\dagger b_i + \frac{1}{2})$$
\[ H^{(1)}_{\text{EV}} = 2^{-1/2} \sum_{\mu=1}^{M} \left( \frac{\partial \epsilon_\mu}{\partial Q_\nu} \right)_0 (b_\mu + b_\mu^\dagger) \delta[a'_\mu a_\mu - n_\mu] + \frac{1}{4} \sum_{\nu, \mu=1}^{M} \left( \frac{\partial^2 \epsilon_\mu}{\partial Q_\nu \partial Q_\nu} \right)_0 (b_\mu + b_\mu^\dagger)(b_\nu + b_\nu^\dagger) \delta[a'_\mu a_\nu + a'_\nu a_\mu] + \frac{1}{2} \sum_{\nu, \mu=1}^{M} \left( \frac{\partial^2 \epsilon_\mu}{\partial Q_\nu^2} \right)_0 (b_\mu + b_\mu^\dagger)(b_\nu + b_\nu^\dagger) \delta[a'_\mu a_\nu + a'_\nu a_\mu] \]

In a diagrammatic method to get function \( G_k(\nu) \) one should calculate the GF for first [1,2,11,12]:
\[ G_k(\nu) = -i\hbar^{-1} \int_{-\infty}^{\infty} dt e^{i\delta_k t} \langle \nu_0 \mid \partial \{ a_k(t) a_k^\dagger(0) \} \mid \nu_0 \rangle \] (8)
and the function \( G_k(\nu) \) can be found from the relation
\[ \pi N_k(\nu) = a \ln G_k(\nu - \eta) \] (9)
Choosing the unperturbed Hamiltonian to be \( H_0 \), one could define the GF as follows:
\[ G^{\theta}_{kk}(t) = 2 \delta_k \exp \left[ -i \hbar^{-1} (E_k + \Delta E) t \right] \cdot \sum_n \left| \langle \eta \mid U(t) \rangle \right|^2 \exp \left( \pm i \hbar \cdot \partial \eta t \right) \] (10)
The direct method for calculation of \( N_k(\nu) \) as the imaginary part of the GF includes a definition of the vertical I.P. (V.I.P.s) of the reference molecule and then of \( N_k(\nu) \). The zeros of the functions

\[ D_k(\nu) = \left[ e^\nu + \Sigma(\nu) \right]_k \] (11)

where \( \left[ e^\nu + \Sigma \right]_k \) denotes the \( k \)-th eigenvalue of the diagonal matrix of the one-particle energies added to matrix of the self-energy part, are the negative V. I. P. ‘s for a given geometry. One can write [2,11,12]:
\[ (V.I.P.)_k = -(\epsilon_k + F_k) \]
\[ F_k = \Sigma_k (-(V.I.P.)_k) = \frac{1}{1 - \delta_{\Sigma_k} \epsilon_k / \partial \epsilon} \Sigma_k (\epsilon_k) \] (12)

Expanding the ionic energy about the equilibrium geometry of the reference molecule in a power series of the normal coordinates of this molecule leads to a set of linear equations in the unknown normal coordinate shifts \( \delta Q_s \) and new coupling constants are then:
\[ g_i = \pm \left( \frac{1}{\sqrt{2}} \right) \left( \delta \epsilon_k F_k \right) / \partial Q_i \] (13)
The coupling constants and are calculated by the well-known perturbation expansion of the self-energy part. In second order one obtains:

\[
\sum \langle j | V_{ij} - V_{jk} | k \rangle \sum \langle k | V_{ij} - V_{kj} | j \rangle + \sum \langle i | V_{ij} - V_{kj} | j \rangle \sum \langle k | V_{ij} - V_{kj} | j \rangle
\]

and the coupling constant \( g \) are written as [17]:

\[
g_i \approx \pm \frac{1}{\sqrt{2}} \frac{\partial \epsilon_i}{\partial \epsilon_i} \sum \langle k | V_{ij} - V_{kj} | j \rangle \sum \langle k | V_{ij} - V_{kj} | j \rangle
\]

The pole strength of the corresponding GF:

\[
\rho_k = \left\{ 1 - \frac{\partial}{\partial \epsilon} \sum \langle k | V_{ij} - V_{kj} | j \rangle \sum \langle k | V_{ij} - V_{kj} | j \rangle \right\}^{-1} ; \rho_k \geq 0, \\
g_i \approx g_i^0 \left[ \rho_k + g_i (\rho_k - 1) \right]
\]

Below we give another the definition of the pole strength corresponding to V. I. P.’s.

3. Fermi-liquid quasiparticle density functional theory

The quasiparticle Fermi-liquid version of the DFT [1-3,8,17] is used to determine the coupling constants etc. The master equations can be obtained on the basis of variational principle, if we start from a Lagrangian of a molecule \( L_q \).

It should be defined as a functional of quasiparticle densities:

\[
\sum \sum_{n} \left[ \nabla \cdot \sum_{r} \left( e_{r} + F_{r} \right) \right]
\]

The densities \( \nu_0 \) and \( \nu_1 \) are similar to the HF electron density and kinetical energy density correspondingly; the density \( \nu_2 \) has no an analog in the HF or DFT theory and appears as result of account for the energy dependence of the mass operator \( \Sigma \). A Lagrangian \( L_q \) can be written as a sum of a free Lagrangian and Lagrangian of interaction: \( L_q = L_q^0 + L_q^{int} \), where the interaction Lagrangian is defined in the form, which is characteristic for a standard DFT (as a sum of the Coulomb and exchange-correlation terms), however, it takes into account for the energy dependence of a mass operator \( \Sigma \):

\[
L_q^{int} = -\sum_{i,k} \frac{1}{2} \sum_{r} \beta_{k} F(r_1, r_2) \nu_i(r_1) \nu_k(r_2) d_1 d_2
\]

where \( F \) is an effective potential of the exchange-correlation interaction. The constants \( \beta_{i} \) are defined in Ref. [8,17]. The single used constant \( \beta_{02} \) can be calculated by analytical way, but it is very useful to remember its connection with a spectroscopic factor \( F_{sp} \) of the system [18]:

\[
F_p = \left\{ 1 - \frac{\partial}{\partial \epsilon} \sum \langle k | V_{ij} - V_{kj} | j \rangle \sum \langle k | V_{ij} - V_{kj} | j \rangle \right\}^{-1}
\]

The terms \( \partial \sum \partial \) and \( \sum \) is directly linked [2,17]. In the terms of the Green function method expression (7) is in fact corresponding to the pole strength of the Green’s function [2]. The new element of an approach is connected with using the DFT correlation functional of the Gunnarsson-Lundqvist, Lee-Yang-Parr (look details in ref. [13-16]).

3. Results and conclusions

In further calculation as potential \( V_X \) we use the exchange-correlation pseudo-potential which contains the correlation (Gunnarsson-Lundqvist) potential and relativistic exchanger Kohn-Sham one [40-42]. As example in table 1 we present our calculational data for spectroscopic factors of some atoms together with available experimental data and results, obtained in the Hartree-Fock theory plus random phase approximation. As an object of studying we choose the diatomic molecule of \( N_2 \) for application of the combined Green’s function method and quasiparticle DFT approach. The nitrogen molecule has been naturally discussed in many papers. The valence V. I. P. of \( N_2 \) have been calculated [1,13,14,24] by the method of Green’s functions and therefore the pole strengths \( p_k \) are known and the mean values
$q_k$ can be estimated. It should be reminded that the $N_2$ molecule is the classical example where the known Koopmans’ theorem (KT) even fails in reproducing the sequence of the V. I. P. ‘s in the PE spectrum. From the HF calculation of Cade et al.[24] one finds that including reorganization the V. I. P. ‘s assigned by and improve while for $\pi$ V. I. P. the good agreement between the Koopmans value and the experimental one is lost, leading to the same sequence as given by Koopmans’ theorem. In Table 1 the experimental V. I. P. ‘s (a), the one-particle HF energies (b), the V. I. P. ‘s calculated by Koopmans’ theorem plus the contribution of reorganization (c), the V. I. P. ‘s calculated with Green’s functions method (d), the combined Green functions and DFT approach (e), the similar our results (f).

### Table 1.
The experimental and calculated V. I. P. (in eV) of $N_2$ ($R_k$ is the contribution of reorganization (see text))

<table>
<thead>
<tr>
<th></th>
<th>Exp $-\varepsilon^b_k$</th>
<th>KT $-\varepsilon^b_k$</th>
<th>EKT $-\varepsilon^b_k$</th>
<th>GF $-\varepsilon^b_k$</th>
<th>MCEP $-\varepsilon^b_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \sigma_g$</td>
<td>15.6</td>
<td>17.24</td>
<td>16.37</td>
<td>15.31</td>
<td>15.52</td>
</tr>
<tr>
<td>$1 \pi_u$</td>
<td>16.98</td>
<td>16.73</td>
<td>16.73</td>
<td>16.80</td>
<td>17.24</td>
</tr>
<tr>
<td>$2 \sigma_u$</td>
<td>18.78</td>
<td>21.13</td>
<td>21.13</td>
<td>19.01</td>
<td>18.56</td>
</tr>
<tr>
<td>Exp - Reorg.</td>
<td>GF+ Reorg.</td>
<td>GF- All corr.</td>
<td>GF- DFT</td>
<td>This work</td>
<td></td>
</tr>
<tr>
<td>$3 \sigma_g$</td>
<td>15.6</td>
<td>16.0</td>
<td>15.50</td>
<td>15.52</td>
<td>15.58</td>
</tr>
<tr>
<td>$1 \pi_u$</td>
<td>16.98</td>
<td>15.7</td>
<td>16.83</td>
<td>16.85</td>
<td>16.93</td>
</tr>
<tr>
<td>$2 \sigma_u$</td>
<td>18.78</td>
<td>19.9</td>
<td>18.59</td>
<td>18.63</td>
<td>18.71</td>
</tr>
</tbody>
</table>

Besides, the comparisons are made in Table 1 with the multiconfigurational electron propagator method (MCEP) and extended KT (EKT) theory (the extended KT has been implemented using multiconfigurational self-consistent field wave functions within different basis sets (I-IV) [52], calculated with the GAMESS, HONDO, and SIRIUS programs. The EKT ionization energies for the 3 and 1 are comparable to the MCEP values. Note that our data are in physically reasonable agreement with the best theoretical values and experimental data. But the most important point of all consideration is connected the principal possibility to reproduce diatomic spectra by applying a one-particle theory with accounting for the correlation and reorganization effects. The combined DFT-GF theoretical approach can be prospectively used for quantitative treating photoelectron spectra of more complicated diatomic molecules.

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Summary

We present the optimized version of the hybrid combined density functional theory (DFT) and the Green’s-functions (GF) approach to quantitative treating the diatomic photoelectron spectra. The Fermi-liquid quasiparticle version of the density functional theory is used. The density of states, which describe the vibrational structure in photoelectron spectra, is defined with the use of combined DFT-GF approach and is well approximated by using only the first order coupling constants in the optimized one-quasiparticle approximation. Using the combined DFT-GF approach leads to significant simplification of the calculation and increasing an accuracy of theoretical prediction.

Key words: photoelectron spectra of molecules, Green’s functions, density functional theory
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ОБОБЩЕННЫЙ МЕТОД ФУНКЦИЙ ГРИНА И ФУНКЦИОНАЛА ПЛОТНОСТИ
В ОПРЕДЕЛЕНИИ КОЛЕБАТЕЛЬНОЙ СТРУКТУРЫ ФОТОЭЛЕКТРОННОГО
СПЕКТРА ДВУХАТОМНЫХ МОЛЕКУЛ

Резюме
Мы представляем оптимизированную версию гибридной комбинированной теории функционала плотности (DFT) и метода функций Грина (GF) для количественного описания фотоэлектронных спектров двухатомных молекул. Используется модель ферми-жидкостная квазичастичная версия теории функционала плотности. Плотность состояний, которые описываются колебательную структуру в фотоэлектронных спектрах, определяется с использованием комбинированного DFT-GF подхода и физически разумно аппроксимируется с использованием только первого порядка констант связи в одноквазичастичном приближении. Использование комбинированного DFT-GF подхода приводит к значительному упрощению молекулярных расчетов и увеличению точности теоретического предсказания.

Ключевые слова: фотоэлектронный спектр молекул, метод функций Грёна, теория функционала плотности

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УДОСКОНАЛЕННЫЙ МЕТОД ФУНКЦИЙ ГРИНА И ФУНКЦИОНАЛА ГУСТИНИ
У ВИЗНАЧЕННІ ВІБРАЦІЙНОЇ СТРУКТУРИ ФОТОЕЛЕКТРОННОГО СПЕКТРУ
ДВОАТОМНИХ МОЛЕКУЛ

Резюме
Мы представляем оптимизированную версию гибридной комбинированной теории функционала густини (DFT) и методу функций Грёна (GF) для кількісного опису фотоелектронних спектрів двохатомних молекул. Використовується фермі-рідинна квазічастична версія теорії функціоналу густини. Густинна стану, яка описує коливальну структуру в фотоелектронних спектрах, визначається з використанням комбінованого DFT-GF підходу та фізично розумно апроксимується за допомогою тільки першого порядку констант зв’язку в одноквазичастинковому наближенні. Використання комбинированного DFT-GF подхода призводить до значного спрощення молекулярних обчислень та збільшення точності теоретичного прогнозування.

Ключові слова: фотоелектронний спектр молекул, метод функцій Грёна, теорія функціонала густини