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A.V. Glushkov[1], *A.A. Kuznetsova*[2]
[1]Odessa State Environmental University, L'vovskaya str.15, Odessa
[2]Odessa National Maritime Academy, Didrikhson str. 4, Odessa
E-mail: glushkovav@gmail.com

RELATIVISTIC ENERGY APPROACH TO ATOMIC SYSTEMS IN A STRONG ELECTROMAGNETIC FIELD IN PLASMAS

The fundamentals of a consistent relativistic approach to determination of radiation and spectral characteristics of the atomic systems (atoms, multicharged ions in plasmas conditions) in a strong external electromagnetic field (EMF), which is based on a relativistic energy formalism (REA) (adiabatic Gell-Mann and Low formalism) and relativistic many-body perturbation theory (RMBPT) formalism for atomic systems in a plasmas are presented. Within an energy approach in relativistic approximation the Gell-Mann and Low formula expresses the imaginary part of an atomic level energy shift δE through the QED scattering matrix, which includes an interaction as with an EMF as with the photon vacuum field (spontaneous radiative decay). It results in possibility of an uniform simultaneous consideration of spontaneous and (or) induced, radiative processes and their interference. The radiation atomic lines position and shape fully determine a spectroscopy of the plasmas atomic systems in an EMF. The effective modified technique, based on the Ivanova-Ivanov method of differential equations, for computing the infinite sums in expressions for a spectral line shifts and broadening is shortly described. The fundamentals of a formalism of the relativistic gauge-invariant RMBPT with the optimized Dirac-Kohn-Sham (DKS) and Debye-Hückel zeroth approximations are presented.

1. Introduction

Spectroscopy of atomic and molecular systems in a strong electromagnetic field belongs to one of the most relevant and practically important directions of modern optics and spectroscopy, in particular, as one of the main sources of spectroscopic information for atomic, molecular, nuclear physics, laser physics, quantum and photo-electronics, plasma physics, astrophysics, etc. In recent years, great interest in both the theoretical fundamental and its numerous applied aspects has grown dramatically. This is due to many reasons, and first of all, we are talking about significant progress in the development of new experimental research methods, in particular, a significant increase in the intensity and quality of laser radiation, the use of accelerators, heavy ion colliders , sources of synchrotron radiation, neutron sources, etc. (e.g.[1-26]), which opens completely new opportunities for the study of increasingly energetic processes, stimulates the development of new theoretical methods for calculating their characteristics in spectroscopy due to collisions of electrons, photons, atoms, molecules. It's appropriate to remind that in many modern plasma experiments (using tokamak plasma, laser plasma, etc.), as well as in astrophysical, space, laboratory, etc. electric fields of various classes are available in the plasma, including quasi-monochromatic electric fields with a relative width of the frequency band, broadband electric fields, finally, the specified environment is influenced by powerful electromagnetic pulses, which as a result induce extremely complex from a theoretical point of view, physical processes in plasma.

In this paper we present the fundamentals of a consistent approach to atomic systems (atoms, multicharged ions in plasmas conditions) in a strong external electromagnetic field (EMF), which is based on a relativistic energy formalism (REA) (adiabatic Gell-Mann and Low formalism) [23-34] and relativistic many-body perturbation theory (RMBPT) formalism for atomic systems in a plasmas (e.g. [35-37]). Within an energy approach in relativistic approximation the Gell-Mann and Low formula expresses the imaginary part of an

atomic level energy shift δE through the QED scattering matrix, which includes an interaction as with an EMF as with the photon vacuum field (spontaneous radiative decay). The fundamentals of a new formalism of the relativistic gauge-invariant RMBPT with the optimized Dirac-Kohn-Sham and Debye-Hückel zeroth approximations are presented.

2. Relativistic energy approach to atomic systems in an electromagnetic field in plasmas

The theoretical basis of the relativistic energy approach in atomic spectroscopy was widely discussed earlier (see, e.g. [21-32]) and here we will focus on the key topics following to Refs. [22,27-29,32]. Let us note that in the theory of the non-relativistic atom a convenient field procedure is known for calculating the energy shifts δE of degenerate states. This procedure is connected with the secular matrix M diagonalization [27,28]. In constructing M, the Gell-Mann and Low adiabatic formula for δE is used. In relativistic version of the Gell-Mann and Low formula δE is connected with electrodynamical scattering matrice, which includes interaction with a laser field. Naturally, in relativistic theory the secular matrix elements are already complex in the second perturbation theory (PT) order. Their imaginary parts are connected with radiation decay possibility. The total energy shift is usually presented in the form [27-29]:

$$\delta E = \mathrm{Re}\delta E + i\mathrm{Im}\delta E, \tag{1a}$$

$$\mathrm{Im}\delta E = -\Gamma / 2, \tag{1b}$$

where Γ is the level width (decay possibility). As it was said, spectroscopy of an atom in a laser field is fully determined by position and shape of the radiation emission and absorption lines. The lines moments M_n are strongly dependent upon the laser pulse quality: intensity and mode constitution [7-13,23,24,27-32]. Let us describe the interaction "atom -EMF" by the following potential:

$$V(r,t) = V(r)\int d\omega f(\omega-\omega_0)\sum_{n=-\infty}^{\infty}\cos(\omega_0 t + \omega_0 n\tau) \tag{2}$$

Here ω_0 is the central EMF radiation frequency, n is the whole number. The potential V represents the infinite duration of EMF pulses with known frequency τ. Next we will consider the interaction of an atom with a single pulse. The function $f(\omega)$ is a Fourier component of the EMF pulse. The condition:

$$\int d\omega f^2(\omega) = 1 \tag{3}$$

normalizes potential $V(r,t)$ on the definite energy in the pulse. It is worth to comment that the EMF pulses with Lorentzian, Gaussian or more complicated shape can be considered. As it was indicated in Ref. [28-32], the main program results in the calculating an imaginary part of energy shift $\mathrm{Im}\delta E_\alpha(\omega_0)$ for any atomic level as the function of the central laser frequency ω_0. An according function has the shape of the resonance, which is connected with the transition α-s (α, s-discrete levels) with absorption (or emission) of the "k" number of photons. We will calculate the following quantities for the multiphoton resonance:

$$\delta\omega(s-\alpha\,|\,k) = \int d\omega \mathrm{Im}\delta E_\alpha(\omega)\ (\omega-\omega_{s\alpha}/k)/N, \tag{4}$$

$$M_n = \int d\omega \mathrm{Im}\delta E_\alpha(\omega)\ (\omega-\varpi_{s\alpha}/k)^n/N, \tag{5}$$

$$N = \int d\omega \mathrm{Im}\delta E_\alpha(\omega)\ , \tag{6}$$

$$\varpi_{s\alpha} = \omega_{s\alpha} + k\cdot\delta\omega(s-\alpha\,|\,k) \tag{7}$$

where N is the normalizing multiplier; $\omega_{s\alpha}$ is position of the non-shifted line for transition s-α, $\delta\omega(s\text{-}\alpha|k)$ is the line shift under k-photon absorption. To determine the quantities M_n , one should need to obtain an expansion of E_α to the following PT series:

$$\delta E_\alpha = \sum \delta E_\alpha^{(2k)}(\omega_0). \tag{8}$$

An external EMF shifts and broadens the atomic levels. The standard quantum approach relates complex eigenenergies $\delta E = \delta E_r + i\Gamma/2$

and complex eigenfunctions to the corresponding resonances [12,24,32]. The field effects drastically increase upon going from one excited level to another. The highest levels could overlap forming a "new continuum" with lowered boundary. In the case of a strong field, its potential should appear in the Dirac equations already in the zeroth-order approximation (the solution is Dirac-Volkov type function). On the other hand, it is convenient to use methods such as operator PT with included well known "distorted-waves" zeroth approximation" in the frame of the formally exact PT.

As usually [32], next one should use the known Gell-Mann and Low adiabatic formula for δE_α with QED scattering matrix. According to [27-32], the representation of the S- matrix in the form of PT series induces the expansion for δE_α:

$$\delta E_\alpha(\omega_0) = \lim_{\gamma\to 0}\gamma \sum_{k_1 k_2 \ldots k_l} c(k_1,k_2,\ldots,k_l) J_\gamma(k_1,k_2,\ldots,k_l), \tag{9}$$

$$J_\gamma(k_1,k_2,\ldots,k_l) = \prod_{j=1} S_\gamma^{(k_j)}, \tag{10}$$

$$S_\gamma^{(n)} = (-1)^n \int_{-\infty}^{0} dt_1 \ldots \int_{-\infty}^{t_{n-1}} dt_n \langle \Psi_\alpha | V_1 V_2 \ldots V_n | \Psi_\alpha \rangle, \tag{11}$$

$$V_i = \exp(iH_0 t_i) V(rt_i) \exp(-iH_0 t_i) \exp(\gamma t_i) \tag{12}$$

where S_γ is QED scattering matrix, γ is an adiabatic parameter, H_0 is the unperturbed atomic Hamiltonian (below it will be defined for atomic systems in the Debye plasmas), $c(k_1, k_2,\ldots,k_n)$ are the numerical coefficients. The details of rather cumbersome transformations are presented in Refs. [32], where the structure of matrix elements $S_\gamma^{(n)}$ is also described. As an example, let us note that $S_\gamma^{(2)}$ can be determined as follows:

$$S_\gamma^{(2)} \sim \int_{-\infty}^{0} dt_1 \int_{-\infty}^{0} dt_2 \exp[\gamma(t_1+t_2)] \langle \Psi_\alpha | \exp(iH_0 t_1) V(rt_1) \cdot$$

$$\cdot \exp(-iH_0 t_1) \exp(iH_0 t_2) V(rt_2) \exp(-iH_0 t_2) | \Psi_\alpha \rangle. \tag{13}$$

After some transformations one can get the expressions for line moments. The final results for quantities (5) for the Gaussian shape laser pulse are as follows:

$$\delta\omega(s-a|k) = [\pi\Delta/(k+1)k][E(s,\omega_{s\alpha}/k) - E(\alpha,\omega_{s\alpha}/k)],$$

$$M_2 = \Delta^2/\mathrm{k}\,,$$

$$M_3 = \{4\pi\Delta^3/[k(k+1)]\}[E(s,\omega_{s\alpha}/k) - E(\alpha,\omega_{s\alpha}/k)], \tag{14}$$

where

$$E(j,\omega_{s\alpha}/k) = \frac{1}{2}\sum_{s_i} V_{js_i} V_{s_i j} \left[\frac{1}{\omega_{js_i} + \omega_{s\alpha}/k} + \frac{1}{\omega_{js_i} + \omega_{s\alpha}/k}\right] \tag{15}$$

The summation in (15) is over all atomic states. The equations (1)-(15) describe the main characteristics of the radiative emission and absorption line of atomic system near the resonant EMF frequency $\omega_{s\alpha}/k$. The corresponding expressions for the Lorentzian shape laser pulse are given in Refs. [23,32]. For the soliton-like pulse it is necessary to use some approximations to simplify the expressions and perform the numerical calculation [32]. The next serious problem is calculation of the sum (15), which includes infinite summations over the complete set of unperturbed (or distorted in the zeroth approximation in a case of a strong EMF) atomic states. One of the most widespread methods for calculating the sums (20) is the Green function method (look below). However, as it was indicated in Refs. [23,25,27-30], the more preferable and effective method is based on the advanced algorithm of differential equations, initially developed by Ivanova-Ivanov [28]. It is worth to note that this method has been frequently used earlier in calculations of different atomic system energy and spectroscopic characteristics (see, e.g., [23-34]). The necessary sums can be expressed through sums of the following one-electron matrix elements:

$$S = \sum_{n_1} \langle n\chi m|V|n_1\chi_1 m_1\rangle \langle n_1\chi_1 m_1|V|n\chi m\rangle / (\varepsilon_{n_1\chi_1 m_1} - \varepsilon)\,, \tag{16}$$

where $(n\mu m)$ – quantum numbers of one-electron states, $\varepsilon = \varepsilon_{n\chi m} + \omega_{s\alpha}/k$ is the energy parameter. One-electron energies $\varepsilon_{n\chi m}$ include the rest energy $(\alpha Z)^{-2}$, where α is the fine structure constant and Z is charge of a nucleus. Here and below we use the Coulomb units (1 C.u.=Z^2 a.u.e.; a.u.e.= 1 atomic unit of energy). For definiteness, let us concretize an

interaction of atom with a EMF. In particular, for the typical dipole interaction the corresponding potential is:

$$V(\mathbf{r})=(\mathbf{a},\boldsymbol{\alpha}), \qquad (17)$$

where **a** is a vector of polarization of radiation; $\boldsymbol{\alpha}$ is a vector of the Dirac matrices.
One should introduce a bi-spinor of the following form [28-32]:

$$\Psi_{\chi_1 m_1}=\sum_{n_1}\phi_{n_1\chi_1 m_1}\left\langle n_1\chi_1 m_1\left|V\right|n\chi m\right\rangle/\left(\varepsilon_{n_1\chi_1 m_1}-\varepsilon\right) \qquad (18)$$

The radial parts *F, G* of a bi-spinor Ψ satisfy the system of the following differential equations [28,32]:

$$-F'/\alpha Z+(1+\chi_1)F/\alpha Zr+A_-G=C\left(\mathbf{a}\left|j_1l_1m_1,\ j\tilde{l}m\right.\right)f_{n\chi}/\alpha Z \qquad (19a)$$

$$G'/\alpha Z+(1-\chi_1)G/\alpha Zr+A_+F=C\left(\mathbf{a}\left|j_1l_1m_1,\ j\tilde{l}m\right.\right)g_{n\chi}/\alpha Z \qquad (19b)$$

$$A_\pm=U_{MF}(r)\pm 1/(\alpha Z)^2-\varepsilon \qquad (20)$$

Here Z is a nuclear charge, α is the fine structure constant, F and G are the components of the bi-spinor (18); $U_{MF}(r)$ is some atomic mean-field potential (see possible expressions, e.g., in Refs. [23-25]). The final expression for the sum (16) can be written as follows:

$$S=\int dr\ r^2\left[f_{n\chi_1}\cdot G(r)\cdot C[(a|jlm,j_1\tilde{l}_1m_1)]\right.$$
$$\left.+g_{n\chi}\cdot F(r)\cdot C\left(a|j\tilde{l}m_1,j_1l_1m_1\right)\right] \qquad (21)$$

The alternative approach to calculating (16) is based on the using method of the Green's function of Dirac equation and presented, for example, in Ref. [32] in the Dirac-Kohn-Sham model of multielectron atom. The Green's function is defined as the solution of the inhomogeneous Dirac equation:

$$\left(\hat{H}-\zeta\right)G_E\left(r_1r_2\right)=\delta\left(r_1-r_2\right), \qquad (22)$$

where $\hat{H}$ is the Dirac Hamiltonian, ζ is an energy parameter. The known spectral decomposition of the Green's function is as follows:

$$G\left(r_1r_2\vee E\right)=\sum_{n\chi m}\Psi_{n\chi m}\left(r_2\right)\Psi_{n\chi m}\left(r_1\right)/\left(E_{n\chi}-E\right) \qquad (23)$$

where one can usually allocate partial contributions with a fixed χ (Dirac's angular quantum number), each of which is a product of the radial $G\left(r_1r_2\vee E,\chi\right)$ and angular parts. In the relativistic theory, the Green's function is a 4-component matrix:

$$G(r_1\,r_2\,|\,E,\chi)=\begin{pmatrix}\hat{F}(r_>)F(r_<) & \hat{F}(r_>)G(r_<)\\ \hat{G}(r_>)F(r_<) & \hat{G}(r_>)G(r_<)\end{pmatrix}, \qquad (24)$$

where $r_\square(r_\square)$ is more (less) of r_1,r_2. According to Ref. [32], the system of the corresponding Dirac equations for F and G component in the Dirac-Kohn-Sham approximation is as follows:

$$F'=-\left(\chi+|\chi|\right)F/r+\tilde{\alpha}\cdot\left[V_N(r)+V^{\mathrm{DKS}}(r)-i\xi-\tilde{\alpha}^{-2}\right]G,$$
$$G'=\left(\chi-|\chi|\right)G/r-\tilde{\alpha}\cdot\left[V_N(r)+V^{\mathrm{DKS}}(r)-i\xi+\tilde{\alpha}^{-2}\right]F, \qquad (25)$$

$$\tilde{F}'=-\left(\chi+|\chi|\right)\tilde{F}/r+\tilde{\alpha}\cdot[V_N(r)+V^{DKS}(r)-i\xi-\tilde{\alpha}^{-2}]\,\tilde{G},$$
$$\tilde{G}'=\left(\chi+|\chi|\right)\tilde{G}/r-\tilde{\alpha}\cdot[V_N(r)+V^{DKS}(r)-i\xi+\tilde{\alpha}^{-2}]\tilde{F}, \qquad (26)$$

where $\tilde{\alpha}=\alpha Z$, $V_N(r)$ is the potential of a nucleus. The functions (*F, G*) represent the first fundamental solution, which is regular for $r\to 0$ and singular for $r\to\infty$. Any combination $\left(\tilde{F},\tilde{G}\right)+Xr^{2|\chi|}(F,G)$ satisfies the above written equations for $\left(\tilde{F},\tilde{G}\right)$ and represents singular solution at zero [29,32]. The right chosen combination $\left(\widetilde{F,G}\right)$ for the single value of the mixing coefficient X (regular for $r\to\infty$) is second fundamental solution $\left(\widetilde{f,g}\right)$. The corresponding condition is as follows:

$$(F,G)\sim\exp(-\mathrm{Ar});,\ A=\left(\tilde{\alpha}^{-2}+\xi^2\tilde{\alpha}^2\right)^{12}. \qquad (27)$$

Other details can be found in Refs. [26-32].

3. Relativistic Debye model for atomic systems in plasmas

Below we shortly present the fundamentals of a relativistic many-body perturbation theory (RMBPT) with the Debye shielding model Hamiltonian for electron-nuclear and electron-electron systems [35-37]. The optimized one-electron representation in the PT zeroth approximation is constructed by means of the correct treating the gauge dependent multielectron contribution of the lowest PT corrections to the radiation widths of atomic levels. Within the known model by Debye and Hückel (e.g. [2-4,35], a plasmas environment effect is modelled by the shielding parameter μ, which describes a shape of the long-rang potential. An interaction potential between two charged particles in a plasmas can me described by the known Yukawa-type potential. A difference between the Yukawa type potential and standard Coulomb potential is in account for the effect of plasma, which is modeled by the shielding parameter. This parameter μ is connected with the plasma parameters such as the temperature T and the charge density n is as follows:

$$\mu \sim \sqrt{e^2 n / k_B T} \qquad (28)$$

Here e is the electron charge and k_B is the Boltzman constant. The density n is given as a sum of the electron density N_e and the ion density N_k of the k-th ion species with the nuclear charge q_k : $n = N_e + \sum_k q_k^2 N_k$. It is very useful to remind the simple estimates for the shielding parameter. For example, under typical laser plasma conditions of T~ 1keV and n~ 10^{23} cm^{-3} the parameter μ is of the order of 0,1 in atomic units. By introducing the Yukawa-type electron-nuclear attraction and electron-electron repulsion potentials, the Debye shielding model Dirac Hamiltonian for electron-nuclear and electron-electron subsystems is given in atomic units as follows (e.g. [35,36]):

$$H = \sum_i [\alpha cp - \beta mc^2 - Z\, exp(-\mu r_i)/r_i] + \sum_{i>j} \frac{(1-\alpha_i \alpha_j)}{r_{ij}} exp(-\mu r_{ij}) \qquad (29)$$

where c is the velocity of light and Z is a charge of the atomic ion nucleus. The formalism of the relativistic many-body PT for atoms in a plasmas is further constructed in the same way as the analogous RMBPT formalism for the free relativistic atomic systems (e.g. [25-32]). However, in the PT zeroth approximation of our RMBPT version we use a mean-field potential, which includes the Yukawa-type potential (insist of the pure Coulomb one) plus exchange Kohn-Sham potential and additionally the Lundqvist-Gunnarson correlation potential (with the optimization parameter b) as in Refs. [16-23,25,32]. As alternative one could use an optimized model potential by Ivanova-Ivanov (for Ne-like ions) [6], which is calibrated within the special ab initio procedure within the relativistic energy approach [28,31]. As usually, a standard field procedure is used for calculating the energy shifts δE of degenerate states of atomic system. This procedure is connected with the secular matrix M diagonalization [27,29]. In constructing M, the Gell-Mann and Low adiabatic formula for δE_α is used. The secular matrix elements are already complex in the PT second order (the first order on the inter-electron interaction). Their imaginary parts are connected with the radiation decay (transition, scattering, ionization etc) possibility. It is important to note that the computing the energies and radiative transition matrix elements is reduced to calculation and the further diagonalization of the complex matrix M .and determination of matrix of the coefficients with eigen state vectors $B^{IK}_{ie,iv}$ [27-29]. To calculate all necessary matrix elements one must use a basis of the one-quasiparticle relativistic functions. Namely this basis is proposed to use in computing radiation spectral lines characteristics of atomic systems in a plasmas. This is a novelty element of a whole theory.

4. Conclusions

To conclude let us note that in this paper we shortly present the fundamentals of a new, consistent relativistic approach to determination of radiation and spectral characteristics of the atomic systems (atoms, multicharged ions in plasmas conditions) in a

strong external EMF, which is based on a adiabatic Gell-Mann and Low formalism and RMBPT formalism for atomic systems in a plasmas. Within an REA the known Gell-Mann and Low formula expresses the imaginary part of an atomic level energy shift δE through the QED scattering matrix, which includes an interaction as with an EMF as with the photon vacuum field (spontaneous radiative decay). The fundamentals of a new formalism of the relativistic gauge-invariant RMBPT with optimized Dirac-Kohn-Sham and Debye-Hückel zeroth approximations and consistent technique for generating an optimized single-quasiparticle representation for atomic systems in the Debye plasmas are presented. It is well known that an adequate description of the radiation and spectral parameters requires using optimized basis's of relativistic wave functions. The approach presented includes an effective ab initio optimization basis construction p[procedure in a natural way. There is used the minimization of the gauge dependent multielectron contribution of the lowest RMBPT corrections to radiation widths of atomic levels, which is determined by imaginary part of an energy shift δE. In the RMBPT higher orders there appear diagrams, whose contribution into ImδE accounts for the polarization effects. This contribution describes collective effects and it is dependent upon electromagnetic potentials gauge (gauge non-invariant contribution δE_{ninv}). This value is considered to be the typical representative of electron correlation effects, whose minimization is a reasonable criteria in the searching for the RMBPT optimal one-electron basis [23-27]. In conclusion, it should be noted the approach presented can be applied to many actual problems modern nuclear physics too as well as astrophysics, laser physics and quantum, photoelectronics (e.g. [5,6,23,38-40]).

References:

1. Glushkov, A.V., Khetselius, O.Y., Maruani, J., Brändas, E. (Eds.) *Advances in Methods and Applications of Quantum Systems in Chemistry, Physics, and Biology. Series: Progress in Theoretical Chem. and Phys.* Cham: Springer. Vol.33, 2021.
2. Oks, E., *Stark Broadening of Hydrogen and Hydrogenlike Spectral Lines in Plasmas: The Physical Insight*. Oxford: Alpha Science Int, 2006.
3. Oks E., Dalimier E., Stamm R., Stehle C., Gonzalez M., Spectral Line Shapes in Plasmas and Gases, ed *Int. J. Spectr*. 2010, *1,* 852581.
4. Griem R., *Spectral Line Broadening by Plasmas*. Academic Press: N-Y., 1974.
5. Bagratashvili V.N., Letokhov V.S., Makarov A.A., Ryabov E.A. *Multiple Photon infrared Laser Photophysics and Photochemistry*. Harwood Acad.Publ.: N.-Y., 1995.
6. Khetselius, O.Yu. *Quantum structure of electroweak interaction in heavy finite Fermi-systems*. Astroprint: Odessa, 2011
7. Delone N., Krainov, V.P., *Multiphoton Processes*; Springer: N.-Y., 1994.
8. Fedorov, M.V. Stabilization of atom in a strong laser field. *Phys. Uspekhi*. 1999, *169(1),* 66—71.
9. Akulin V.M., Karlov N.V., *Intense resonant interactions in quantum electronics*, Nauka: Moscow, 1987
10. Mittleman, M. *Introduction to the Theory of Laser-Atom Interactions;* Plenum Press: N.Y., 1982.
11. *Atoms in Intense Laser Fields*, Gavrila, M., Ed.; Acad. Press: San Diego, 1992.
12. Glushkov A.V., Ivanov, L.N. DC strong-field Stark effect: consistent quantum-mechanical approach. *J. Phys. B: At. Mol. Opt. Phys.* 1993, *26,* L379-386.
13. Kulander, K.C.; Lewenstein, M. Multiphoton and Strong-Field Processes. *In Atomic, Molecular, & Optical Physics Handbook*, Drake, G. W. F., Ed.; AIP Press: New York, 1996, pp 828-838.
14. Khetselius, O.Yu. *Hyperfine structure of atomic spectra*. Astroprint: 2008.
15. Khetselius, O.Yu. Relativistic perturbation theory calculation of the hyperfine structure parameters for some heavy-element isotopes. *Int. Journ. Quant. Chem.* 2009, *109,* 3330-3335.
16. Khetselius, O.Yu. Relativistic calculation of the hyperfine structure parameters for heavy elements and laser detection of the heavy isotopes. *Phys.Scripta.* 2009, *135,*

014023.
17. Khetselius, O.Yu. Optimized relativistic many-body perturbation theory calculation of wavelengths and oscillator strengths for Li-like multicharged ions. Adv. Quant. Chem. 2019, *78*, 223-251.
18. Svinarenko, A. Study of spectra for lanthanides atoms with relativistic many-body perturbation theory: Rydberg resonances. *J. Phys.: Conf. Ser.* 2014, *548,* 012039.
19. Florko, T.A., Tkach, T.B., Ambrosov, S.V., Svinarenko, A.A. Collisional shift of the heavy atoms hyperfine lines in an atmosphere of the inert gas. *J. Phys.: Conf. Ser*. 2012, *397*, 012037.
20. Grant I. *Relativistic Quantum Theory of Atoms and Molecules*. Oxford Univ. Press: Oxford, 2007.
21. Ivanova, E.P.; Grant, I.P. Oscillator strength anomalies in the neon isoelectronic sequence with applications to x-ray laser modeling. *J. Phys.B: At. Mol. Opt. Phys.* 1998, *31,* 2871-2880.
22. Ivanov, L.N.; Letokhov, V.S. Selective ionization of atoms by a light and electric field. *Quant. Electr*.(in Russian) 1975*, 2*, 585-590.
23. Glushkov, A.V. *Relativistic Quantum theory. Quantum mechanics of atomic systems*. Astroprint: Odessa, 2008.
24. Glushkov, A.V. *Atom in an electromagnetic field*. KNT: Kiev, 2005.
25. Glushkov, A.V. *Relativistic and Correlation Effects in Spectra of Atomic Systems*. Astroprint: 2006.
26. Ivanova, E., Glushkov, A. Theoretical investigation of spectra of multicharged ions of F-like and Ne-like isoelectronic sequences. J. Quant. Spectr. and Rad. Tr. 1986, *36*(2), 127-145.
27. Ivanova, E.P., Ivanov, L.N., Glushkov, A., Kramida, A. High order corrections in the relativistic perturbation theory with the model zeroth approximation, Mg-Like and Ne-Like Ions. *Phys. Scripta* 1985, *32,* 513-522.
28. Glushkov, A., Ivanov, L. Radiation decay of atomic states: atomic residue polarization and gauge noninvariant contributions. *Phys.Lett.A.*1992, *170,* 33
29. Ivanov, L.N., Ivanova, E.P., Knight, L. Energy approach to consistent QED theory for calculation of electron-collision strengths: Ne-like ions. *Phys. Rev. A.* 1993, *48,* 4365-4374
30. Glushkov, A.V. Energy approach to resonance states of compound superheavy nucleus and EPPP in heavy nuclei collisions. In Low Energy Antiproton Physics. AIP: New York, *AIP Conf. Proc*. 2005, *796*, 206-210
31. Glushkov, A.V. Advanced relativistic energy approach to radiative decay processes in multielectron atoms and multicharged ions. In: *K.Nishikawa, J. Maruani, E. Brändas*, *G. Delgado-Barrio, P.Piecuch (Eds) Quantum Systems in Chemistry and Physics*: *Progress in Methods and Applications. Series: Progress in Theoretical Chem. and Phys.* Dordrecht: Springer. 2012, *26*, 231–252.
32. Glushkov, A. Multiphoton spectroscopy of atoms and nuclei in a laser field: Relativistic energy approach and radiate-on atomic lines moments method. *Adv. in Quantum Chem.* 2019, *78*, 253-285.
33. Glushkov, A.V., Operator Perturbation Theory for Atomic Systems in a Strong DC Electric Field. In: *M. Hotokka, J.Maruani, E. Brändas„ G.Delgado-Barrio (Eds) Advances in Quantum Methods and Applications in Chemistry, Physics, and Biology. Series: Progress in Theoretical Chem. and Phys.* Cham: Springer. 2013, *27*, 161–177.
34. Glushkov, A. Spectroscopy of coope-rative muon-gamma-nuclear processes: Energy and spectral parameters *J. Phys.: Conf. Ser.* 2012, *397,* 012011.
35. Malinovskaya S., Glushkov A., Khetselius O.,Lopatkin Yu., Loboda A., Svinarenko A., Nikola L., Perelygina T., Generalized energy approach to calculating electron collision cross-sections for multicharged ions in a plasma: Debye shielding model. Int. J. Quant. Chem. 2011, *111*(2), 288-296.
36. Glushkov A, Buyadzhi V, Svinarenko A, Ternovsky E., Advanced relativistic energy approach in electron-collisional spectroscopy of multicharged ions in plasma. In: *Y.A. Wang, M. Thachuk, R. Krems, J.Maruani, Concepts, Methods and Applications of Quantum Systems in*

Chemistry and Physics. Series: Progress in Theoretical Chem. and Phys. Cham: Springer. 2018, *31*, 55-69.
37. Ternovsky E., Relativistic spectroscopy of multicharged ions in plasmas: Li-like ions. *Photoelectr*. 2019. *28*, 105-112.
38. Khetselius, O.Y., Spectroscopy of cooperative electron-gamma-nuclear processes in heavy atoms: NEET effect *J. Phys.: Conf. Ser.* 2012, *397,* 012012.
39. Khetselius, O.Yu. Relativistic Energy Approach to Cooperative Electron-γ-Nuclear Processes: NEET Effect. In: *K.Nishikawa, J. Maruani, E. Brändas*, *G. Delgado-Barrio, P.Piecuch (Eds) Quantum Systems in Chemistry and Physics*: *Progress in Methods and Applications. Series: Progress in Theoretical Chem. and Phys.* Dordrecht: Springer. 2012, *26,* 217-229.
40. Dubrovskaya, Yu., Khetselius, O.Yu., Vitavetskaya, L., Ternovsky, V.B., Serga, I.N., Quantum chemistry and spectroscopy of pionic atomic systems with accounting for relativistic, radiative, and strong interaction effects. *Adv. Quantum Chem.* 2019, *78*, 193-222

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Glushkov A.V., Kuznetsova A.A.

RELATIVISTIC ENERGY APPROACH TO ATOMIC SYSTEMS IN A STRONG ELECTROMAGNETIC FIELD IN PLASMAS

Summary. The fundamentals of a consistent relativistic approach to determination of radiation and spectral characteristics of the atomic systems (atoms, multicharged ions in plasmas conditions) in a strong external electromagnetic field (EMF), which is based on a relativistic energy formalism adiabatic Gell-Mann and Low formalism) and relativistic many-body perturbation theory (RMBPT) formalism for atomic systems in a plasmas are presented. Within an energy approach in relativistic approximation the Gell-Mann and Low formula expresses the imaginary part of an atomic level energy shift δE through the QED scattering matrix, which includes an interaction as with an EMF as with the photon vacuum field (spontaneous radiative decay). It results in possibility of an uniform simultaneous consideration of spontaneous and (or) induced, radiative processes and their interference. The radiation atomic lines position and shape fully determine a spectroscopy of the plasmas atomic systems in an EMF. The effective modified technique, based on the Ivanova-Ivanov method of differential equations, for computing the infinite sums in expressions for a spectral line shifts and broadening is shortly described. The fundamentals of a new formalism of the relativistic gauge-invariant RMBPT with the optimized Dirac-Kohn-Sham and Debye-Hückel zeroth approximations and consistent technique for generating an optimized single-quasiparticle representation for atomic systems in the Debye plasmas are presented.

Keywords: Relativistic energy approach, many-body perturbation theory, atomic systems, plasmas environment, electromagnetic field,

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Глушков О.В., Кузнецова Г.О.

РЕЛЯТИВІСТСЬКИЙ ЕНЕРГЕТИЧНИЙ ПІДХІД ДО АТОМНИХ СИСТЕМ У СИЛЬНОМУ ЕЛЕКТРОМАГНІТНОМУ ПОЛІ В ПЛАЗМІ

Резюме. Розроблені основи послідовного підходу до визначення енергетичних, радіаційних та спектроскопічних характеристик атомних систем (атомів, багатозарядних іонів у плазмових умовах) у сильному зовнішньому електромагнітному полі (ЕМП), який базується на релятивістському енергетичному формалізмі (адіабатичний формалізм Гелл-Мана та Лоу) і релятивістській багаточастинковій теорії збурень (RMBPT) для атомних систем у плазмі. У рамках релятивістського енергетичного підходу формула Гелл-Манна і Лоу виражає уявну частину зсуву енергії атомного рівня через КЕД матрицю розсіювання, яка включає взаємодію як з ЕМП, так і з фотонним вакуумом (останнє відповідає за спонтанний радіаційний розпад в системі). В межах розробленої теорії є можливим послідовне одночасне урахування спонтанних і (або) індукованих радіаційних процесів та їх інтерференції. Положення і форма атомних ліній випромінювання повністю визначають спектроскопію атомних систем в плазми у зовнішньому ЕМП. Описано ефективну модифіковану техніку, засновану на методі диференціальних рівнянь Іванової-Іванова, для обчислення нескінченних сум у виразах для зсуву та уширення спектральних ліній, та (або) виразів, що відповідають другому порядку RMBPT. Представлені основи нового формалізму релятивістської калібрувально-інваріантної RMBPT з оптимізованими нульовим наближенням Дірака-Кона-Шема та Дебая-Хюккеля і визначено послідовну схему генерації оптимізованого одноквазічастинкового представлення для атомних систем у плазмовому середовищу.

Ключові слова: релятивістський енергетичний підхід, релятивістська багаточастинкова теорія збурень, атомні системи, плазмове середовище, електромагнітне поле

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