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## ADVANCED DATA FOR HYDROGEN ATOM IN CROSSED ELECTRIC AND MAGNETIC FIELDS

Spectroscopy of atoms in the crossed external electric and magnetic fields is investigated on the basis of the operator perturbation theory. As a novel element within the operator perturbation theory, we use more flexible functions for model function, which imitates an electric field. In a case of the crossed electric and magnetic fields we develop more effective finite differences numerical scheme. As illustration, some advanced data for the hydrogen atom in the electric and crossed external electric and magnetic fields are listed. Advanced data for hydrogen atom are listed.

### 1. Introduction

From the standard quantum mechanics it is well known that the external electric field shifts and broadens the bound state atomic levels. One should note that the usual quantum-mechanical approach relates complex eigen-energies (EE)  $E = E_r + 0,5iG$  and complex eigen-functions (EF) to the shape resonances [1-6]. The calculation difficulties in the standard quantum mechanical approach are well known and described in many Refs. Let us remind that the usual quasiclassical WKB approximation overcomes these difficulties for the states, lying far from “new continuum” boundary and, as rule, is applied in the case of a relatively weak electric field. The same is regarding the widespread asymptotic phase method (c.f.[2]). Quite another calculation procedures are used in the Borel summation of the divergent perturbation theory (PT) series and in the numerical solution of the difference equations following from expansion of the wave function over finite basis [2,3,9,10].

Experimental observation of the Stark effect in a constant (DC) electric field near threshold in hydrogen and alkali atoms led to the discovery of resonances extending into the ionization continuum (c.f.[1]). Calculation of the characteristics of these resonances as well as the Stark resonances in the strong electric field and crossed electric and magnetic fields remains very

important problem of as modern atomic physics [1-20].

In this paper we go on our studying of spectroscopy of atoms in the crossed external electric and magnetic fields. Our method of studying is based on the known formalism of the operator perturbation theory (OPT) [1-3]. According to [1-5], the essence of operator perturbation theory approach is the inclusion of the well known method of “distorted waves approximation” in the frame of the formally exact perturbation theory. As a novel element within the operator perturbation theory, we use more flexible functions for model function, which imitates an electric field. In a case of the crossed electric and magnetic fields we develop more effective finite differences numerical scheme. As illustration, some advanced data for the hydrogen atom in the electric and crossed external electric and magnetic fields are listed.

### 2. Method of operator perturbation theory

As our approach to strong field DC Stark effect was presented in a series of papers (see, for example, [1-6]), here we are limited only by the key aspects. According to [2,3], the Schrödinger equation for the electronic eigen-function taking into account the uniform DC electric field (the field strength is  $F$ ) and the field of the nucleus (Coulomb units are used: a unit is  $\hbar^2/Ze^2 m$  and a

unit of  $mZ^2 e^4 / h^2$  for energy) looks like:

$$[-(1 - N/Z) / r + Fz - 0,5\Delta - E] \psi = 0 \quad (1)$$

where  $E$  is the electronic energy,  $Z$  — charge of nucleus,  $N$  — the number of electrons in atomic core. Our approach allow to use more adequate forms for the core potential (c.f.[25-27]). According to standard quantum defect theory (c.f.[3]), relation between quantum defect value  $\mu_p$ , electron energy  $E$  and principal quantum number  $n$  is:  $\mu_p = n - z^* (-2E)^{-1/2}$ . As it is known, in an electric field all the electron states can be classified due to quantum numbers:  $n, n_1, n_2, m$  (principal, parabolic, azimuthal:  $n = n_1 + n_2 + m + 1$ ). Then the quantum defect in the parabolic co-ordinates  $\delta(n_1, n_2, m)$  is connected with the quantum defect value of the free ( $F=0$ ) atom by the following relation [3]:

$$\delta(n_1, n_2, m) = (1/n) \sum_{l=m}^{n-1} (2l+1) (C_{J, M-m; lm}^{JM})^2 \mu_l$$

$$J = (n-1)/2, \quad M = (n_1 - n_2 + m)/2;$$

After separation of variables, equation (1) in parabolic co-ordinates could be transformed to the system of two equations for the functions  $f$  and  $g$ :

$$f'' + \frac{|m|+1}{t} f' + [0,5E + (\beta_1 - N/Z) / t - 0,25 F(t) / t] f = 0 \quad (2)$$

$$g'' + \frac{|m|+1}{t} g' + [0,5E + \beta_2 / t + 0,25 F(t) / t] g = 0 \quad (3)$$

coupled through the constraint on the separation constants:  $\beta_1 + \beta_2 = 1$ .

For the uniform electric field  $F(t) = F$ . In ref. [11], the uniform electric field  $\varepsilon$  in (3) and (4) was substituted by model function  $F(t)$  with parameter  $\tau$  ( $\tau = 1.5 t_2$ ). To simplify the calculation procedure, the uniform electric field  $\varepsilon$  in (3) and (4) should be substituted by the function [57,58]:

$$\varepsilon(t) = \frac{1}{t} \varepsilon \left[ (t - \tau) \frac{\tau^4}{\tau^4 + t^4} + \tau \right] \quad (4)$$

th sufficiently large  $\tau$  ( $\tau = 1.5 t_2$ ). The function  $\varepsilon(t)$

practically coincides with the constant  $\varepsilon$  in the inner barrier motion region ( $t < t_2$ ) and disappears at  $t \gg t_2$ . Potential energy in equation (4) has the barrier. Two turning points for the classical motion along the  $\eta$  axis,  $t_1$  and  $t_2$ , at a given energy  $E$  are the solutions of the quadratic equation ( $\beta = \beta_1, E = E_0$ ). According to [1-3], one should know two zeroth order EF of the  $H_0$ : bound state function  $\Psi_{Eb}(\varepsilon, \nu, \varphi)$  and scattering state function  $\Psi_{Es}(\varepsilon, \eta, \varphi)$  with the same EE in order to calculate the width  $G$  of the concrete quasi-stationary state in the lowest PT order. Firstly, one would have to define the EE of the expected bound state. It is the well known problem of states quantification in the case of the penetrable barrier. Further one should solve the system (2, 3) system with the total Hamiltonian  $H$  using the conditions [11]:

$$f(t) \rightarrow 0 \text{ at } t \Rightarrow \infty$$

$$\text{with} \quad \partial x(\beta, E) / \partial E = 0 \quad (5)$$

$$x(\beta, E) = \lim_{t \Rightarrow \infty} [g^2(t) + \{g'(t) / k\}^2] t^{|m|+1}.$$

These two conditions quantify the bounding energy  $E$ , with separation constant  $\beta_1$ . The further procedure for this two-dimensional eigenvalue problem results in solving of the system of the ordinary differential equations(2, 3) with probe pairs of  $E, \beta_1$ . The bound state EE, eigenvalue  $\beta_1$  and EF for the zero order Hamiltonian  $H_0$  coincide with those for the total Hamiltonian  $H$  at  $\varepsilon \Rightarrow 0$ , where all the states can be classified due to quantum numbers:  $n, n_1, l, m$  (principal, parabolic, azimuthal) that are connected with  $E, \beta_1, m$  by the well known expressions.. The scattering states' functions must be orthogonal to the above defined bound state functions and to each other. According to the OPT ideology [11,12], the following form of  $g_{Es}'$  is possible:

$$g_{Es}(t) = g_1(t) - z_2' g_2(t) \quad (6)$$

with  $f_{Es}$ , and  $g_1(t)$  satisfying the differential equations (2) and (3). The function  $g_2(t)$  satisfies the non-homogeneous differential equation, which differs from (3) only by the right hand term, disappearing at  $t \Rightarrow \infty$ .

In Ref, [7] it has been presented approach, based on solution of the 2-dimensional Schrödinger equation for an atomic system in crossed fields and operator perturbation theory. For definiteness, we consider a dynamics of the complex non-coulomb atomic systems in a static magnetic and electric fields. The hamiltonian of the multi-electron atom in a static magnetic and electric fields is (in atomic units) as follows:

$$H = 1/2(p_\rho^2 + l_z^2 / \rho^2) + B_z / 2 + (1/8)B^2 \rho^2 + (1/2)p_z^2 + F + V(r) \quad (7)$$

where the electric field  $F$  and magnetic field  $B$  are taken along the  $z$ -axis in a cylindrical system; In atomic units:  $1 \text{ a.u.} B = 2.35 \times 10^5 \text{ T}$ ,  $1 \text{ a.u.} F = 5,144 \times 10^6 \text{ kV/cm}$ . For solution of the Schrödinger equation with hamiltonian equations (7) we constructed the finite differences scheme which is in some aspects similar to method [7]. An infinite region is exchanged by a rectangular region:  $0 < \rho < L_\rho$ ,  $0 < z < L_z$ . It has sufficiently large size; inside it a rectangular uniform grid with steps  $h_\rho$ ,  $h_z$  was constructed. The external boundary condition, as usually, is:  $(\partial \Psi / \partial n)_r = 0$ .

The knowledge of the asymptotic behaviour of wave function in the infinity allows to get numerical estimates for  $L_\rho$ ,  $L_z$ . A wave function has an asymptotic of the kind as:  $\exp[-(-2E)^{1/2}r]$ , where  $(-E)$  is the ionization energy from stationary state to lowest Landau level. Then  $L$  can be estimated as  $L \sim 9(-2E)^{-1/2}$ . The more exact estimate is found empirically. The finite-difference scheme is constructed as follows. The three-point symmetric differences scheme is used for second derivative on  $z$ . The derivatives on  $\rho$  are approximated by  $(2m+1)$ -point symmetric differences scheme with the use of the Lagrange interpolation formula differentiation. To calculate the values of the width  $G$  for resonances in atomic spectra in an electric field and crossed electric and magnetic field one can use the modified operator perturbation theory method (see details in ref.[10,20]). Note that the imaginary part of the state energy in the lowest PT order is:

$$\text{Im}E = G/2 = \pi \langle \Psi_{Eb} | H | \Psi_{Es} \rangle^2 \quad (8)$$

with the total Hamiltonian of system in an electric and magnetic field. The state functions  $\Psi_{Eb}$  and  $\Psi_{Es}$  are assumed to be normalized to unity and by the  $\delta(k - k')$ -condition, accordingly. Other calculation details can be found in ref. [7]. Different application are considered in Refs. [21-57].

### 3. Illustration results and conclusion

As an illustration,, we make computing the energy of the ground state of the hydrogen atom in crossed fields and compare results with data obtained within analytical perturbation theory by TurbinerV (see. [8]) for the case of sufficiently weak fields. Table 1 shows the values of the energy of the ground state of the hydrogen atom (the following designations:  $E+E^{\parallel}$  - energy for the case of the electric and magnetic fields are parallel;  $E+E^{\perp}$  corresponds to the case of the electric and magnetic fields are perpendicular).

Table 1  
Energy values (Ry) of the H ground state in electric  $F$  ( $1 \text{ au} = 5.14 \cdot 10^9 \text{ V/cm}$ ) and magnetic  $B$  ( $1 \text{ au.} B = 2.35 \cdot 10^5 \text{ T}$ ) fields

$F, B$ $10^{-2}$	$E+E^{\parallel}$ Turbiner theory [8]	$E+E^{\perp}$ [5]
0,0	-1,000000	-1,000000
0,1	-1,000004	-1,000004
0,5	-1,000099	-1,000099
1,0	-1,000402	-1,000401
1,5	-1,000906	-1,000905
2,0	-1,001617	-1,001616
2,5	-1,002542	-1,002540
3,0	-1,003685	-1,003682
3,5	-1,005054	-1,005053
4,0	-1,0066619	-1,006659
4,5	-1,008520	-1,008517
5,0	-1,010642	-1,010636
$F, B$ $10^{-2}$	$E+E^{\parallel}$ This work	$E+E^{\perp}$ This work
0,0	-1,000000	-1,000000
0,1	-1,000004	-1,000004
0,5	-1,000100	-1,000099
1,0	-1,000402	-1,000401
1,5	-1,000906	-1,000905
2,0	-1,001617	-1,001616
2,5	-1,002541	-1,002535
3,0	-1,003684	-1,003673
3,5	-1,005054	-1,005036
4,0	-1,006686	-1,006627
4,5	-1,008519	-1,008464
5,0	-1,010638	-1,010556

Since the considered electric field is sufficiently weak, difference between all data in Table 1 is quite little. At the same time it is clear that the perturbation theory in the standard quantum-mechanical version is correct exclusively for the weak fields, while for strong fields it can lead to substantially inaccurate data. Really, in Table 2 we list the results for the Stark resonances energies and widths of the ground state hydrogen atom in the DC electric field with the strength  $\epsilon=0.1$  and  $0.8$  a.u., obtained within the most exact alternative methods and our data (see [2]).

Table 2  
**The energies and widths of the Stark resonances of the H ground state ( $F=0.1, 0.8$  a.u.). Notation: (A) Hehenberger, H.V. McIntosh and E. Brändas, (B) Farrelly and Reinhardt, (C) Rao, Liu and Li [18], (D) Glushkov-Ivanov, the standard OPT method; (E)- Popov et al; (F) – our data**

$F$ , a.u.	Method	$E_r$ , a.u.	$\Gamma/2$ , a.u.
0.10	A	-0.52743	$0.725 \times 10^{-2}$
	C	-0.527418	$0.7269 \times 10^{-2}$
	D	-0.527419	$0.2269 \times 10^{-2}$
	E	-0.527	$0.227 \times 10^{-2}$
	F	-0.527418	$0.7269 \times 10^{-2}$
0.80	B	-0.6304	0.5023
	C	-0.630415	0.50232
	D	-0.630416	0.50232
	F	-0.630415	0.50231

The comparison of our data (Table 2: F) with earlier similar results, obtained within the summation of divergent PT series, the numerical solution with expansion of the wave function over finite basis, a complex scaling plus B-spline calculation, the standard OPT one (Table 2: A-E) shows quite acceptable agreement. We believe that the OPT method with new elements will be especially efficient for atoms in the strong crossed electric and magnetic fields, where the standard methods (usual perturbation theory etc) deal with great principal and computational problems).

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## ADVANCED DATA FOR HYDROGEN ATOM IN CROSSED ELECTRIC AND MAGNETIC FIELDS

### Summary

Spectroscopy of atoms in the crossed external electric and magnetic fields is investigated on the basis of the operator perturbation theory. As a novel element within the operator perturbation theory, we use more flexible functions for model function, which imitates an electric field. In a case of the crossed electric and magnetic fields we develop more effective finite differences numerical scheme. As illustration, some advanced data for the hydrogen atom in the electric and crossed external electric and magnetic fields are listed. Advanced data for hydrogen atom are listed.

**Key words:** atom, hydrogen, crossed electric and magnetic fields

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## АТОМ ВОДОРОДА В СКРЕЩЕННЫХ ЭЛЕКТРИЧЕСКОМ И МАГНИТНОМ ПОЛЯХ

### Резюме

Работа посвящена изучению спектроскопических параметров атомов в постоянном электрическом и скрещенных электрическом и магнитном полях на основе формализма известной операторной теории возмущений. В качестве нового элемента в операторную теорию возмущений оператора вводится применение более эффективной функции для модельной функции, имитирующей электрическое поле. В случае скрещенных электрического и магнитного полей разработана более эффективная численная конечно-разностная схема. В качестве иллюстрации приведены некоторые уточненные данные для атома водорода в сильном электрическом поле и скрещенных электрическом и магнитном полях. Приведены численные данные для атома водорода.

**Ключевые слова:** атом, водород, скрещенные электрическое и магнитное поля

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## АТОМ ВОДНЮ В СХРЕЩЕНИХ ЕЛЕКТРИЧНОМУ І МАГНІТНОМУ ПОЛЯХ

### Резюме

Робота присвячена вивченню спектроскопічних параметрів атомів у сталому електричному та схрещених електричному та магнітному полях на основі відомої операторної теорії збурень. В якості нового елементу в операторну теорію збурень вводиться використання більш ефективної функції для модельної функції, яка імітує зовнішнє електричне поле. У випадку схрещених електричного та магнітного полей розроблена ефективна чисельна скінченно-різницева схема. В якості ілюстрації наведені уточнені данні для атома водню в сильному електричному полі і схрещених електричному та магнітному полях. Наведено чисельні дані для атома водню.

**Ключові слова:** атом, водень, схрещені електричне і магнітне поля