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CHAOTIC DYNAMICS OF RELATIVISTIC BACKWARD-WAVE TUBE WITH ACCOUNTING FOR SPACE CHARGE FIELD AND DISSIPATION EFFECTS: NEW EFFECTS

We have performed an advanced modelling nonlinear dynamics elements for relativistic backward-wave tube (RBWT) with accounting for dissipation and space charge field effects etc. The temporal dependences of the normalized field amplitude (power) in a wide range of variation of the controlling parameters (electric length of an interaction space N, bifurcation parameter L and relativistic factor γ_0) are computed. The dynamic and topological invariants of the RBWT dynamics in auto-modulation and chaotic regimes such as correlation dimensions values, embedding, Kaplan-York dimensions, Lyapunov's exponents, Kolmogorov entropy etc are calculated. It has been discovered the "beak" effect on the plane of parameters: bifurcation Piers-like parameter L – relativistic factor γ_0.

1. Introduction

Powerful generators of chaotic oscillations of microwave range of interest for radar, plasma heating in fusion devices, modern systems of information transmission using dynamic chaos and other applications. Among the most studied of vacuum electronic devices with complex dynamics are backward-wave tubes (BWT) , for which the possibility of generating chaotic oscillations has been theoretically and experimentally found [1-12]. The BWT is an electronic device for generating electromagnetic vibrations of the superhigh frequencies range. Authors [7] formally considered the possible chaos scenario in a single relativistic BWT. Authors [4,6] have studied dynamics of a non-relativistic BWT, in particular, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and the same characteristics of two non-relativistic backward-wave tubes. The authors of [4,6] have solved the equations of nonstationary nonlinear theory for the O-type BWT without account of the spatial charge, relativistic effects, energy losses etc. It has been shown that the finite-dimension strange attractor is responsible for chaotic regimes in the BWT. The multiple studies [1-12], increasing the beam current in the system implemented complex pattern of alternation of regular and chaotic regimes of generation, completes the transition to a highly irregular wideband chaotic oscillations with sufficiently uniform continuous spectrum.

In this work we have performed an advanced modelling emission spectrum and nonlinear dynamics elements for relativistic backward-wave tube (RBWT) with accounting for dissipation and space charge effects etc. The temporal dependences of the normalized field amplitude (power) in a wide range of variation of the controlling parameters (electric length of an interaction space N, bifurcation parameter L and relativistic factor γ_0) are computed. The dynamic and topological invariants of the RBWT dynamics in automodulation and chaotic regimes such as correlation dimensions values, embedding, Kaplan-York dimensions, Lyapunov's exponents, Kolmogorov entropy etc are calculated. WE discovered discovered the «beak» effect on the plane $L – \gamma_0$.

2. Relativistic model and some results

As the key ideas of our technique for nonlinear analysis of chaotic systems have been in details presented in refs. [13-28], here we are limited only by a short representation. We use the standard non-stationary theory [3-7], however, despite the above cited papers we take into account a number of effects, namely, influence of space charge, dissipation, the waves reflections at the ends of the system and others (a modification of model of Refs. [12,13]).

The relativistic dynamics is described system of equations for unidimensional relativistic electron phase $\theta(\zeta,\tau,\theta_0)$ (which moves in the interaction space with phase θ_0 ($\theta_0 \in [0;\ 2\pi]$) and has a coordinate ζ at time moment τ) and field unidimensional complex amplitude

$$\partial^2\theta/\partial\zeta^2 = -L^2\gamma_0^3[(1+\frac{1}{2\pi N}\partial\theta/\partial\zeta)^2 - \beta_0^2]^{3/2}$$

$$\mathrm{Re}[F\exp(i\theta) + \frac{4QC}{ik}\sum_{k=1}^{M} I_k \exp(ik\theta)]$$

$$\partial F/\partial\tau - \partial F/\partial\zeta + dF = -L\widetilde{I}\ ,$$

$$I_k = -\frac{1}{\pi}\int_0^{2\pi} e^{-ik\theta}d\theta_0 \qquad (1)$$

with the corresponding boundary and initial conditions. The dynamical system studied has several controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: i) electric length of an interaction space N; ii) bifurcation parameter $L = 2\pi CN/\gamma_0$ (here C- is the known Piers parameter) ; iii) relativistic factor, which is determined as:

. (2)

As input parameters there were taken following initial values: relativistic factor γ_0=1.5 (further we will increase γ_0 in 2 and 4 times), electrical length of the interaction space $N = k_0 l / (\pi$ =10, electrons speed v_0=0.75c, v_{rp}=0.25c, dissipation parameter D = 5Db, starting reflection parameters: s = 0.5, ρ=0.7, 0< <2π . A choice of φ due to the fact that the dependence upon it is periodic. The influence of reflections leads to the fact that bifurcational parameter L begins to be dependent on the phase φ of the reflection parameter (see discussion regarding it in [7,8]).

The basic idea of the construction of our approach to prediction of chaotic properties of complex systems is in the use of the traditional concept of a compact geometric att (CGA) in which evolves the measurement data, plus the neural networks (NNW) algorithm implementation [10-16]. Let us consider some scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is the start time, Δt is the time step, and n is the number of the measurements. The main task is to reconstruct phase space using as well as possible information contained in $s(n)$. To do it, the method of using time-delay coordinates by Packard et al [17] can be used. The direct using lagged variables $s(n+\tau)$ (here τ is some integer to be defined) results in a coordinate system where a structure of orbits in phase space can be captured. A set of time lags is used to create a vector in d dimensions, $\mathbf{y}(n)$= [$s(n)$, $s(n + \tau)$, $s(n + 2\tau)$, .., $s(n + (d-1)\tau)$], the required coordinates are provided. Here the dimension d is the embedding dimension, d_E. To determine the proper time lag at the beginning one should use the known method of the linear autocorrelation function (ACF) $C_L(\delta)$ and look for that time lag where $C_L(\delta)$ first passes through 0 [4]. The alternative additional approach is provided by the average mutual information (AMI) method as an approach with so called nonlinear concept of independence. The further next step is to determine the embedding dimension, d_E, and correspondingly to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. The dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$. To reconstruct the attractor dimension and to study the signatures of chaos in a time series, one could use such methods as the correlation integral algorithm (CIA) by Grassberger and Procaccia [21] or the false nearest neighbours (FNN) method by Kennel et al [18]. The principal question of studying any complex chaotic system is to build the corresponding prediction model and define how predictable is a chaotic system. The new element of our approach is using the NNW algorithm in forecasting nonlinear dynamics of chaotic systems [9,10]. In terms of the neuro-informatics and neural networks theory the process of modelling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations. Imitating the further evolution of a system within NNW simulation with the corresponding elements of the self-study, self- adaptation, etc., it becomes possible to significantly improve the prediction of its evolutionary dynamics. The fundamental parameters to be computed are the Kolmogorov entropy (and correspondingly the predictability measure as it can be estimated by the Kolmogorov entropy), the Lyapunov's exponents (LE), the Kaplan-Yorke dimension (KYD) etc. The LE are usually defined as asymptotic average rates and they are related to the eigenvalues of the linearized dynamics across the attractor. Naturally, the knowledge of the whole LE allows to determine other important invariants such as the Kolmogorov entropy and the attractor's dimension. The Kolmogorov entropy is determined by the sum of the positive LE. The estimate of the dimension of the attractor is provided by the Kaplan and Yorke conjecture

$\sum$, where j is such that $\sum$

and $\sum$, and the LE are taken in descending order. In Figure 1 we present the flowchart of the our combined chaos-geometric and NNW computational approach to nonlinear analysis and prediction of dynamics of any complex system [10-30].

In figure 2 we list the data on the temporal dependence of normalized field amplitude $F(æ;ô) = \tilde{E}/(2â_0UC^2)$ (our data subject dissipation, the influence of space charge, the effect of reflections waves) at the values of the bifurcation parameter L:(a) – 3.5, (b) – 3.9 (other parameters: γ_0=1.5, 10, s=0.5, ρ=0.7, =1.3π).

I. General analysis of the dynamical problem, processing dynamical variable series for studied complex system (preliminary general analysis of dynamics, evolutionary differential equations treating,…)

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II. Chaos-geometric method: assessment of the presence of chaos:

1. The Gottwald-Melbourne test: $K \rightarrow 1$ – chaos;
2. Fourier decompositions, irregular nature of change – chaos;
3. Spectral analysis, Energy spectra statistics, the Wigner distribution, the spectrum of power, "Spectral rigidity";

⇓

III. The geometry of phase space. Fractal Geometry:

4. Computation time delay τ using ACF or AMI;
5. Determining embedding dimension d_E by the CIA method or FNN points;
6. Calculation multi-fractal spectra. Wavelet analysis;

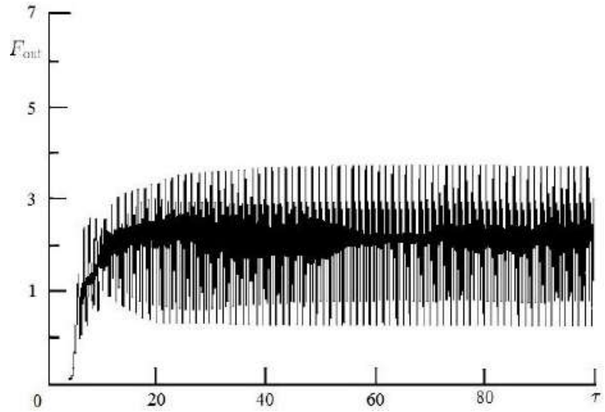
⇓

IV. Prediction model:

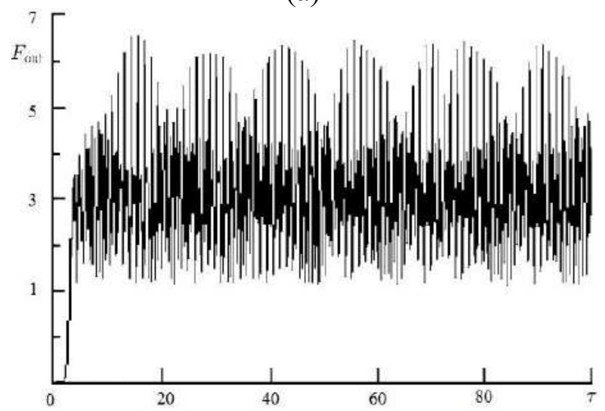
7. Computing global LE: λ_α; KYD d_L, average predictability measure Pr_{max};
8. Determining the number of FNN points for the best prediction results;
9. Methods of nonlinear prediction: nonlinear parameterized function; NNW; optimized trajectories (propagators) algorithms, ...;

Figure 1. Flowchart of the combined chaos-geometric approach and NNW to nonlinear analysis and prediction of chaotic dynamics of the complex systems (devices)

Figures 1a,b are corresponding to the regimes of periodical automodulation (a) and hyper chaotic regime (b). It is worth to note that our results obtained without accounting for the reflection effect are very well correlated with the data by Ryskin-Titov in Ref. [7], where it has been in details studied the RBWT dynamics with.



(a)



(b)

Figure 2. Data on the time dependence of normalized field amplitude F(ζ,τ)(our data with accounting dissipation, the influence of space charge and an effect of wave reflections) at the values of the bifurcation parameter L: (a) 3.0 (b) 4.0 (other parameters: γ_0=1.5, 10, s=0.5, ρ=0.7, =1.3π).

In table 1 we list our data on the correlation dimension d_2, embedding dimension, determined on the basis of false nearest neighbours algorithm (d_N) with percentage of false neighbours (%). calculated for different values of lag τ (data on fig1b, regime of a chaos).

In Table 2 we list our computing data on the Lyapunov exponents (LE), the dimension of the Kaplan-York attractor, the Kolmogorov entropy K_{entr}.

Table 1.

Correlation dimension d_2, embedding dimension, determined on the basis of false nearest neighbours algorithm (d_N) with percentage of false neighbours (%) calculated for different values of lag τ

τ	d_2	(d_N)
60	8.2	10 (12)
8	6.5	8 (2.1)
10	6.5	8 (2.1)

Table 2.

The Lyapunov exponents (LE), the dimension of the Kaplan-York attractor, the Kolmogorov entropy K_{entr}. (our data)

λ_1	λ_2	λ_3	λ_4	K
0.508	0.196	-0.0001	–0.0003	0.704

For studied series there are the positive and negative LE values.The resulting KYD in both cases are very similar to the correlation dimension (calculated by the algorithm by Grassberger-Procachia). More important is the analysis of the RBWT nonlinear dynamics in the plane "relativistic factor – bifurcation parameter."

The numerical solution has shown that under the realistic values of the dissipation parameter, the effect is reduced to the shift of the value of the bifurcation parameter L towards the increase. The most interesting, in our opinion, is the results of the analysis of the change of the nonlinear dynamics of the considered RBWT in the plane "relativistic factor - bifurcation parameter". In this aspect, in fact, the three parametric nonlinear dynamics of the RBWT are fundamentally different from the dynamics of processes in the non-relativistic BWT. In Figure 3 we refer to our calculated diagram which quantitatively shows the limits of automodulation (line I) on the plane of parameters: L-γ_0. Note that line II limits the region where the particle rotation takes place, that is, the used theoretical model (1) works. A characteristic feature of Figure 3 is the presence of the "beak" effect, which, depending on the relativistic factor, goes far into the domain of automodulation.

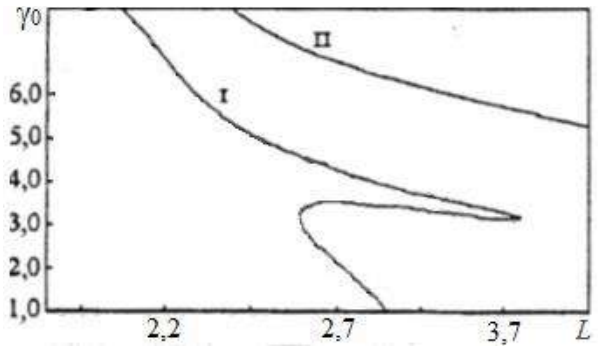


Figure 3. The boundaries of automodulation (line I) on the plane of parameters:L-g_0.

3. Conclusions

In this work we have performed an advanced modelling and for the first time forecasting an emission spectrum and nonlinear dynamics elements for relativistic backward-wave tube (RBWT) with accounting for dissipation and space charge effects etc. The temporal dependences of the normalized field amplitude (power) in a wide range of variation of the controlling parameters (electric length of an interaction space N, bifurcation parameter L and relativistic factor γ_0) are computed. The dynamic and topological invariants of the RBWT dynamics in auto-modulation and chaotic regimes such as correlation dimensions values, embedding, Kaplan-York dimensions, Lyapunov's exponents, Kolmogorov entropy etc are calculated. diagram which quantitatively shows the limits of self-modulation (line I) on the plane of parameters:L-γ_0 .is calculated. It has been discovered the "beak" effect (on the plane of parameters L, γ_0), which, depending on the relativistic factor, goes far into the domain of automodulation for the RBWT studied.

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Summary.

We have performed an advanced modelling and for the first time forecasting an emission spectrum and nonlinear dynamics elements for relativistic backward-wave tube (RBWT) with accounting for dissipation and space charge effects etc. The temporal dependences of the normalized field amplitude (power) in a wide range of variation of the controlling parameters (electric length of an interaction space N, bifurcation parameter L and relativistic factor γ_0) are computed. The dynamic and topological invariants of the RBWT dynamics in auto-modulation and chaotic regimes such as correlation dimensions values, embedding, Kaplan-York dimensions, Lyapunov's exponents, Kolmogorov entropy etc are calculated. It has been discovered the «beak» effect on the plane of parameters: bifurcation Piers-like parameter L – relativistic factor γ_0, which, depending on the relativistic factor, goes far into the domain of automodulation.

Key words: relativistic backward-wave tube, chaos, non-linear methods

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А. В. Глушков, А. В. Цудик, Д. А. Новак, О. В. Дубровский

ХАОТИЧЕСКАЯ ДИНАМИКА РЕЛЯТИВИСТСКОЙ ЛАМПЫ ОБРАТНОЙ ВОЛНЫ С УЧЕТОМ ВЛИЯНИЯ ПОЛЯ ПРОСТРАНСТВЕННОГО ЗАРЯДА И ДИССИПАЦИИ: НОВЫЕ ЭФФЕКТЫ

Резюме.

Представлены результаты моделирования элементов нелинейной динамики для релятивистской обратной волны (РЛОВ) с учетом эффектов диссипации и поля пространственного заряда и др. Временные зависимости нормированной амплитуды поля (мощности) вычислены в широком диапазоне вариаций управляющих параметров (электрическая длина пространства взаимодействия N, параметр бифуркации L и релятивистский фактор γ_0). Рассчитаны динамические и топологические инварианты динамики РЛОВ в автомодуляционном и хаотичном режимах, в частности, значения корреляционной размерности, размерности вложения, Каплана-Йорка, показатели Ляпунова, энтропия Колмогорова и др. Обнаружен эффект «клюва» на плоскости параметров: бифуркационный параметр L - релятивистский фактор γ_0.

Ключевые слова: релятивистская лампы обратной волны, хаос, нелинейные методы

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О. В. Глушков, А. В. Цудік, Д. А. Новак, О. В. Дубровський

ХАОТИЧНА ДИНАМІКА РЕЛЯТИВІСТСЬКОЇ ЛАМПИ ЗВЕРНЕНОЇ ХВИЛІ З УРАХУВАННЯМ ВПЛИВУ ПОЛЯ ПРОСТОРОВОГО ЗАРЯДУ ТА ДИСИПАЦІЇ: НОВІ ЕФЕКТИ

Резюме.

Представлені результати моделювання спектру випромінювання та елементів нелінійної динаміки для релятивістської зворотної хвилі (РЛЗХ) з урахуванням ефектів дисипації та поля просторового заряду тощо. Часові залежності нормированної амплітуди поля (потужності) обчислені в широкому діапазоні варіацій керуючих параметрів (електрична довжина простору взаємодії N, параметр біфуркації L і релятивістський фактор γ_0). Розраховані динамічні та топологічні інваріанти динаміки РЛЗХ в автомодуляційному та хаотичному режимах, зокрема, значення кореляційної розмірності, розмірності вкладення, Каплана-Йорка, показники Ляпунова, ентропія Колмогорова тощо. Виявлено ефект «дзьоба» на площині параметрів: біфуркаційний параметр L - релятивістський фактор γ_0.

Ключові слова: релятивістська лампи зворотної хвилі, хаос, нелінійні методи