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NONLINEAR DYNAMICS OF RELATIVISTIC BACKWARD-WAVE TUBE IN AUTOMODULATION AND CHAOTIC REGIME WITH ACCOUNTING THE EFFECTS WAVES REFLECTION, SPACE CHARGE FIELD AND DISSIPATION

It has been performed quantitative modelling, analysis, forecasting dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects, dissipation, a presence of space charge etc. There are computed the temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N, bifurcation parameter L and relativistic factor γ0. The computed temporal dependence of the field amplitude (power) are very well correlated with the results by Ryskin-Titov, who give the detailed studying the RBWT dynamics with accounting the reflection effect, but without accounting dissipation effect and space charge field influence etc. The analysis techniques including multi-fractal approach, methods of correlation integral, false nearest neighbour, Lyapunov exponent's, surrogate data, is applied analysis of numerical parameters of chaotic dynamics of RBWT. There are computed the dynamic and topological invariants of the RBWT dynamics in auto-modulation(AUM)/chaotic regimes, correlation dimensions values), embedding, Kaplan-York dimensions, Lyapunov's exponents (LE: +,+) Kolmogorov entropy.

1. Introduction

The backward-wave tube is an electronic device for generating electromagnetic vibrations of the superhigh frequencies range. In refs.[1-14] there have been presented the temporal dependences of the output signal amplitude, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and for developed chaos at large values of the dimensionless length parameter. The authors of [1-14] solved the different versions of system of equations of nonstationary nonlinear theory for the O type backward-wave tubes with and without account of the spatial charge, without energy losses etc. It has been shown that the finite-dimension strange attractor is responsible for chaotic regimes in the backward-wave tube.

In our work it has been performed quantitative modelling, analysis, forecasting dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects (g_0>1), dissipation, a presence of a space charge field etc. There are computed the temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N, bifurcation parameter L one and relativistic factor g_0. The computed temporal dependence of the field amplitude (power) are very well correlated with the results by Ryskin-Titov [7], who give the detailed studying the RBWT dynamics with accounting the reflection effect, but without accounting dissipation effect and space charge field influence etc.

2. Method and Results

As the key ideas of our technique for nonlinear analysis of chaotic systems have been in details presented in refs. [13-28], here we are limited only by brief representation. The first important step is a choice of the model of the RBWT dynamics. We use the standard non-stationary theory [3-7], however, despite the cited papers we take into account a number of effects, namely, influence of space charge, dissipation, the waves reflections at the ends of the system and others [12,13]. Usually relativistic dynamics is described system of equations for unidimensional relativistic electron phase $è(æ ô, è_0)$ (which moves in the interaction space with phase q_0 (q_0Î[0; 2p]) and has a coordinate z at time moment t) and field unidimensional complex amplitude $F(æ ô) = \tilde{E} / (2â_0 UC^2)$ as [240, 249]:

$$\partial^2\theta / \partial\zeta^2 = -L^2\gamma_0^3 [\ 1 + \frac{1}{2\pi N}\partial\theta / \partial\zeta)^2 - \beta_0^2\ ]^{3/2}$$

$$\mathrm{Re}[F\exp(i\theta) + \frac{4Q}{k}\sum_{k=1}^{M} I_k \exp(k\ \theta)]$$

$$\partial F / \partial\tau - \partial F / \partial\zeta + dF \ = -L\widetilde{I}\ ,$$

$$I_k = -\frac{1}{ð}\int_0^{ð} e^{-ikè} dè_0 \qquad (1)$$

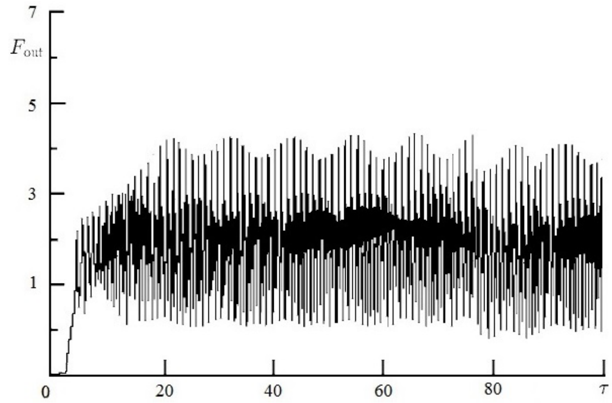
with the corresponding boundary and initial conditions. In is important to note that the system studied has a few controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N, bifurcation parameter $L = 2\pi CN / \gamma_0$ (here C- is the known Piers parameter) and relativistic factor $\gamma_0 = (1-\beta_0^2)^{-1./2}$. As input parameters there were taken following initial values: relativistic factor g_0=1.5 (further we will increase g_0 in 2 and 4 times), electrical length of the interaction space $N = k_0 l / (2\pi)$=10, electrons speed v_0=0.75c, v_{rp}=0.25c, dissipation parameter D = 5Db, starting reflection parameters: s = 0.5, r=0.7, 0<ϕ<2p . A choice of j due to the fact that the dependence upon it is periodic. The influence of reflections leads to the fact that bifurcational parameter L begins to be dependent on the phase j of the reflection parameter (see discussion regarding it in [7,8]).

Since processes resulting in the chaotic behaviour are fundamentally multivariate, it is necessary to reconstruct phase space using as well as possible information contained in the dynamical parameter *s*(*n*), where *n* the number of the measurements. Such a reconstruction results in a certain set of *d*-dimensional vectors y(*n*) replacing the scalar measurements. Packard et al. [19] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables *s*(*n* + t), where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector in *d* dimensions, y(*n*) = [*s*(*n*), *s*(*n* + t), *s*(*n* + 2t),…,*s*(*n* + (*d*-1)t)], the required coordinates are provided. In a nonlinear system, the *s*(*n* + *j*t) are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension *d* is called the embedding dimension, d_E. According to Mañé and Takens [24,25], any time lag will be acceptable is not terribly useful for extracting physics from data. The autocorrelation function and average mutual information can be applied here. The first approach is to compute the linear autocorrelation function C_L(d) and to look for that time lag where C_L(d) first passes through zero (see [18]). This gives a good hint of choice for t at that *s*(*n* + *j*t) and *s*(*n* + (*j* + 1)t) are linearly independent. a time series under consideration have an *n*-dimensional Gaussian distribution, these statistics are theoretically equivalent (see [15]). The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [5,6,18]. First, many of computations for extracting interesting properties from the data require searches and other operations in R^d whose computational cost rises exponentially with *d*. Second, but more significant from the physical point of view, in the presence of noise or other high-D contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal. There are several standard approaches to reconstruct the attractor dimension (see, e.g., [3-6,15]). The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, *C*(*r*), to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [10] is the most commonly used approach. If the time series is characterized by an attractor, then the integral *C*(*r*) is related to the radius *r* as

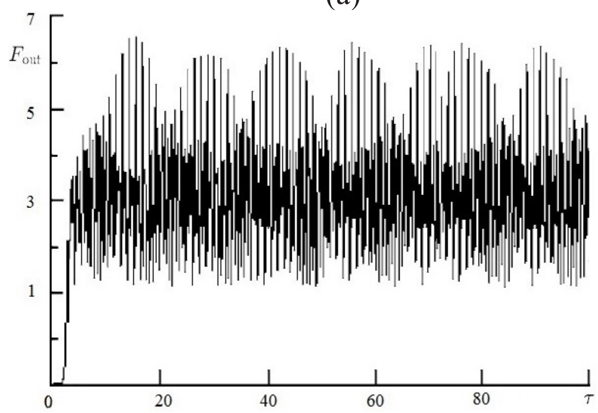
$$d = \lim_{\substack{r\to 0 \\ N\to\infty}} \frac{\log C(r)}{\log r}, \quad (2)$$

where *d* is correlation exponent.The saturation value of correlation exponent is defined as the correlation dimension (d_2) of attractor. The Lyapunov exponents are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local Lyapunov exponents. Since the Lyapunov exponents are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of Lyapunov exponents, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, *K*, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive Lyapunov exponents. There are several approaches to computing the Lyapunov exponents (see, e.g., [5,6,15-18]).

In figure 1 we list the data on the time dependence of normalized field amplitude $F(æ,ô)=\tilde{E}/(2â_0UC^2)$ (our data subject dissipation, the influence of space charge, the effect of reflections waves) at the values of the bifurcation parameter L:(a) – 3.5, (b) – 3.9 (other parameters: g_0=1.5, $N=10$, s=0.5, r=0.7, ϕ=1.3p).



(a)



(b)

Figure 1. Data on the time dependence of normalized field amplitude F(z,t)(our data with accounting dissipation, the influence of space charge and an effect of wave reflections) at the values of the bifurcation parameter L: (a) – 3.5, (b) – 3.9 (other parameters: g_0=1.5, $N=10$, s=0.5, r=0.7, ϕ=1.3p).

Figures 1a,b are corresponding to the regimes of quasi-periodical automodulation (a) and essentially chaotic regime (b). Importantly, our results obtained are very well correlated with the results by Ryskin-Titov in Ref. [7], where it has been in details studied the RBWT dynamics with accounting the reflection effect, but without accounting dissipation effect and space charge field influence etc. In table 1 we list our data on the correlation dimension d_2, embedding dimension, determined on the basis of false nearest neighbours algorithm (d_N) with percentage of false neighbours (%). calculated for different values of lag t (data on fig1b, regime of a chaos).

Table 1.

Correlation dimension d_2, embedding dimension, determined on the basis of false nearest neighbours algorithm (d_N) with percentage of false neighbours (%) calculated for different values of lag t

τ	τ	d_2	(d_N)
60	68	8.1	10 (12)
6	9	6.4	8 (2.1)
8	12	6.4	8 (2.1)

In Table 2 we list our computing data on the Lyapunov exponents (LE), the dimension of the Kaplan-York attractor, the Kolmogorov entropy K_{entr}.

Table 2.

The Lyapunov exponents (LE), the dimension of the Kaplan-York attractor, the Kolmogorov entropy K_{entr}. (our data)

λ_1	λ_2	λ_3	λ_4	K
0.507	0.198	-0.0001	–0.0003	0.71

For studied series there are the positive and negative LE values. he resulting dimension Kaplan York in both cases are very similar to the correlation dimension (calculated by the algorithm by Grassberger-Procachia). More important is the analysis of the RBWT nonlinear dynamics in the plane "relativistic factor – bifurcation parameter." Actually in this context a three-parametric relativistic nonlinear dynamics is fundamentally different from processes in non-relativistic BWT dynamics.

Conclusions

In this work we have performed quantitative modelling, analysis, forecasting dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects (g_0>1), dissipation, a presence of space charge, reflection of waves at the end of deceleration system etc. There are computed the temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N, bifurcation parameter L (the automodulation and chaotic regimes) relativistic factor g_0=1.5-6.0). There are computed the dynamic and topological invariants of the RBWT dynamics in auto-modulation/chaotic regimes, correlation dimensions values, embedding, Kaplan-York dimensions, LE(LE:+,+) Kolmogorov entropy. In the further work we will try to present the bifurcation diagrams with definition of the dynamics self-modulation/chaotic areas in planes: «L-g_0», «D-L», predict emergence of highly-d chaotic attractor, which evolves at a much complicated scenario.

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NON-LINEAR DYNAMICS OF RELATIVISTIC BACKWARD-WAVE TUBE IN SELF-MODULATION AND CHAOTIC REGIME WITH ACCOUNTING THE WAVES REFLECTION, SPACE CHARGE FIELD AND DISSIPATION EFFECTS

Abstract

It has been performed quantitative modelling, analysis of dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects, dissipation, a presence of space charge etc. There are computed the temporal dependences of the normalized field amplitudes in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N, bifurcation parameter L and relativistic factor $\gamma 0$. The computed temporal dependence of the field amplitude is in a good agreement with theoretical data by Ryskin-Titov regarding the RBWT dynamics with accounting the reflection effect, but without accounting dissipation effect and space charge field influence etc. The analysis techniques including multi-fractal approach, methods of correlation integral, false nearest neighbour, Lyapunov exponent's, surrogate data, is applied analysis of numerical parameters of chaotic dynamics of RBWT. There are computed the dynamic and topological invariants of the RBWT dynamics in auto-modulation, chaotic regimes, correlation dimensions values), embedding, Kaplan-York dimensions, LE(+,+) Kolmogorov entropy.

Key words: relativistic backward-wave tube, chaos, non-linear methods

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НЕЛИНЕЙНАЯ ДИНАМИКА РЕЛЯТИВИСТСКОЙ ЛАМПЫ ОБРАТНОЙ ВОЛНЫ В АВТОМОДУЛЯЦИОННОМ И ХАОТИЧЕСКОМ РЕЖИМАХ С УЧЕТОМ ЭФФЕКТОВ ОТРАЖЕНИЯ ВОЛН, ВЛИЯНИЯ ПОЛЯ ПРОСТРАНСТВЕННОГО ЗАРЯДА И ДИССИПАЦИИ

Резюме

Приведены результаты моделирования, анализа динамики процессов в релятивистской лампе обратной волны (РЛОВ) с учета релятивистских эффектов, диссипации, наличия пространственного заряда и т.д. Вычислены временные зависимости нормированной амплитуды поля в широком диапазоне изменения управляющих параметров: электрическая длина пространства взаимодействия N, бифуркационный параметр L и релятивистский фактор $\gamma 0$. Вычисленная зависимость амплитуды поля находится в хорошем согласии с теоретическими данными Рыскина-Титова о динамике РЛОВ с учетом эффекта отражения волн, но без учета эффектов диссипации и влияния поля пространственного заряда, т.д. Техника нелинейного

анализа, которая включает методы корреляционных интегралов, ложных ближайших соседей, экспонент Ляпунова, суррогатных данных, использована для анализа численных параметров хаотического режима в РЛОВ. Рассчитаны динамические и топологические инварианты динамики РЛОВ в автомодуляционном и хаотическом режимах, корреляционная размерность, размерности вложения, Каплан-Йорка, показатели Ляпунова (+, +), энтропия Колмогорова.
Ключевые слова: релятивистская лампы обратной волны, хаос, нелинейные методы

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НЕЛІНІЙНА ДИНАМІКА РЕЛЯТИВІСТСЬКОЇ ЛАМПИ ЗВЕРНЕНОЇ ХВИЛІ В АВТОМОДУЛЯЦІЙНОМУ ТА ХАОТИЧНОМУ РЕЖИМАХ З УРАХУВАННЯМ ЕФЕКТІВ ВІДДЗЕРКАЛЕННЯ ХВИЛЬ, ВПЛИВУ ПОЛЯ ПРОСТОРОВОГО ЗАРЯДУ І ДИСИПАЦІЇ

Резюме

Наведені результати моделювання, аналізу динаміки процесів в релятивістської лампі зворотної хвилі (РЛЗХ) з урахуванням релятивістських ефектів, дисипації, наявності просторового заряду і т.і. Обчислені часові залежності нормованої амплітуди поля в широкому діапазоні зміни керуючих параметрів: електрична довжина простору взаємодії N, біфуркаційний параметр L, і релятивістський фактор $\gamma 0$. Обчислена залежність амплітуди поля знаходиться в хорошій згоді з теоретичними даними Рискіна-Титова щодо динаміки РЛЗХ з урахуванням ефекту віддзеркалення хвиль, але без урахування ефектів дисипації і впливу поля просторового заряду, тощо. Техніка нелінійного аналізу, яка включає методи кореляційних інтегралів, хибних найближчих сусідів, експонент Ляпунова, сурогатних даних, використана для аналізу чисельних параметрів хаотичних режимів у РЛЗХ. Розраховані динамічні та топологічні інваріанти динаміки РЛЗХ в автомодуляціоному і хаотичному режимах, кореляційна розмірність, розмірності вкладення, Каплан-Йорка, показники Ляпунова (+, +), ентропія Колмогорова.
Ключові слова: релятивістська лампи зворотної хвилі, хаос, нелінійні методи