UDC 541.13

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NEW NONLINEAR ANALYSIS, CHAOS THEORY AND INFORMATION TECHNOLOGY APPROACH TO STUDYING DYNAMICS OF THE THE ERBIUM ONE-RING FIBRE LASER

Within new non-linear analysis, chaos theory and information technology approach it is nu-merically investigated chaos dynamics generation in the erbium one-ring fibre laser (EDFL, 20.9mV strength, λ= 1550.190nm) with the control parameters: the modulation frequency f and dc bias voltage of the electro-optical modulator. It is shown that in depending upon f, V values there are realized 1-period f = 75MHz, V = 10V and f = 60MHz, V = 4V), 2-period (f = 68 MHz, V = 10V or f = 60MHz, V = 6V), chaotic (f = 64MHz, V = 10 V and f = 60MHz, V = 10V) regimes; there are calculated LE, correlation, embedding, Kaplan-York dimensions, Kolmogorov entropy and theoretically shown that chaos in the erbium fiber laser device is generated via intermittency by increasing the DC bias voltage and period-doubling bifurcation by reducing the modulation fre-quency.

1 Introduction

It is very known that a chaos is alternative of randomness and occurs in very simple deterministic systems. Although chaos theory places fundamental limitations for long-rage prediction (see e.g. [1-9]), it can be used for short-range prediction since ex facte random data can contain simple deterministic relationships with only a few degrees of freedom. Chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. The past decade has witnessed a large number of studies employing the ideas gained from the science of chaos to characterize, model, and predict the dynamics of various systems phenomena (see e.g. [1-13]). The outcomes of such studies are very encouraging, as they not only revealed that the dynamics of the apparently irregular phenomena could be understood from a chaotic deterministic point of view but also reported very good predictions using such an approach for different systems.

In a modern quantum electronics and laser physics etc there are many systems and devices (such as multi-element semiconductors and gas lasers etc), dynamics of which can exhibit chaotic behaviour. These systems can be considered in the first approximation as a grid of autogenerators (quantum generators), coupled by different way [2,14,15]. In this paper we present an application of a new and advanced known non-linear analysis, chaos theory and information technology methods [1-20] to studying non-linear dynamics of the erbium one-ring fibre laser (EDFL, 20.9mV strength, λ= 1550.190nm) with the control parameters: the modulation frequency *f* and dc bias voltage of the electro-optical modulator. Technique of non-linear analysis includes a whole sets of new algorithms and advanced known methods such as the wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent’s (LE) analysis, and surrogate data method, neural networks prediction approach etc (see details in Refs. [1-34]).

2. Methods of studying dynamics of the laser systems

As used non-linear analysis, chaos theory and information technology methods to studying non-linear dynamics of the laser systems have been earlier in details presented [1-20] here we are limited only by the key ideas. As usually, we formally consider scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is the start time, Δt is the time step, and is n the number of the measurements. Packard et al. [18] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n + \tau)$, where τ is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. First approach to compute τ is based on the linear autocorrelation function. The second method is an approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows [5,7,13]. One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. If a time series under consideration have an n-dimensional Gaussian distribution, these statistics are theoretically equivalent as it is shown in Ref. [22].

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. There are several standard approaches to reconstruct the attractor dimension (see, e.g., [1,7,23]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [23] is the most commonly used approach. According to this algorithm, the correlation integral is

$$C(r) = \lim_{N\to\infty} \frac{2}{N(n-1)} \sum_{\substack{i,j \\ (1\le i<j\le N)}} H\left(r - |\,\mathbf{y}_i - \mathbf{y}_j\,|\right) \tag{1}$$

where H is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for u □ 0, r is the radius of sphere centered on $\mathbf{y}_i$ or $\mathbf{y}_j$, and N is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r\to 0 \\ N\to\infty}} \frac{\log C(r)}{\log r}, \tag{2}$$

where d is correlation exponent that can be determined as the slop of line in the coordinates $\log C(r)$ versus $\log r$ by a least-squares fit of a straight line over a certain range of r, called the scaling region.

There are certain important limitations in the use of the correlation integral analysis in the search for chaos. To verify the results obtained by the correlation integral analysis, we use surrogate data method. The method of surrogate data [1,7,19] is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data. Advanced version is presented in [7-9].

The next step is computing the Lyapunov's exponents (LE). The LE are the dynamical invariants of the nonlinear system. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of LE is considered a measure of the effect of perturbing the initial conditions of a dynamical system. Note that both positive and negative LE can coexist in a dissipative system, which is then chaotic. Since the LE are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of the LE, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, K, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive LE. The inverse of the Kolmogorov entropy is equal to an average predictability.

Estimate of dimension of the attractor is provided by the Kaplan and Yorke conjecture. There are a few approaches to computing the LE.

One of them computes the whole spectrum and is based on the Jacobi matrix of system [27]. In the case where only observations are given and the system function is unknown, the matrix has to be estimated from the data. In this case, all the suggested methods approximate the matrix by fitting a local map to a sufficient number of nearby points.

In our work we use the method with the linear fitted map proposed by Sano and Sawada [27] added by the neural networks algorithm [7-10]. To calculate the spectrum of the LE from the amplitude level data, one could determine the time delay τ and embed the data in the four-dimensional space.

3. Chaotic elements in dynamics of the erbium one-ring fibre laser: Some illustrations and conclussions

Here we present results of the quantitative studying a chaotic dynamics in the erbium one-ring fibre laser with the control parameters: the modulation frequency *f* and dc bias voltage of the electro-optical modulator. Feng and et al. [35] have observed experimentally generate chaos in dynamics in the erbium one-ring fibre laser (laser parameters: the initial strength of 20.9 mV, 1550.190 nm wavelength) with added electro-optical modulator made from crystal LiNbO3.

In the first series of measurements (Exp.1) the constant bias voltage was maintained at 10V, the frequency modulation control parameter *f* was *f* = 64-75MHz. In figure 1 there are listed the measured time-series of the output voltage V_{out} dependence on the frequency modulation by Feng and et al. [35] : Up fig. - *f* = 75 MHz (1-period state), Middle fig.- *f* = 68MHz (2-period state), Down fig.- *f* = 64MHz (chaos).

In a second series of measurements (Exp 2.) by Feng and et al. [35] the frequency modulation was kept at the value of 60 MHz, and the constant bias voltage V was varied from 4V till 10V.

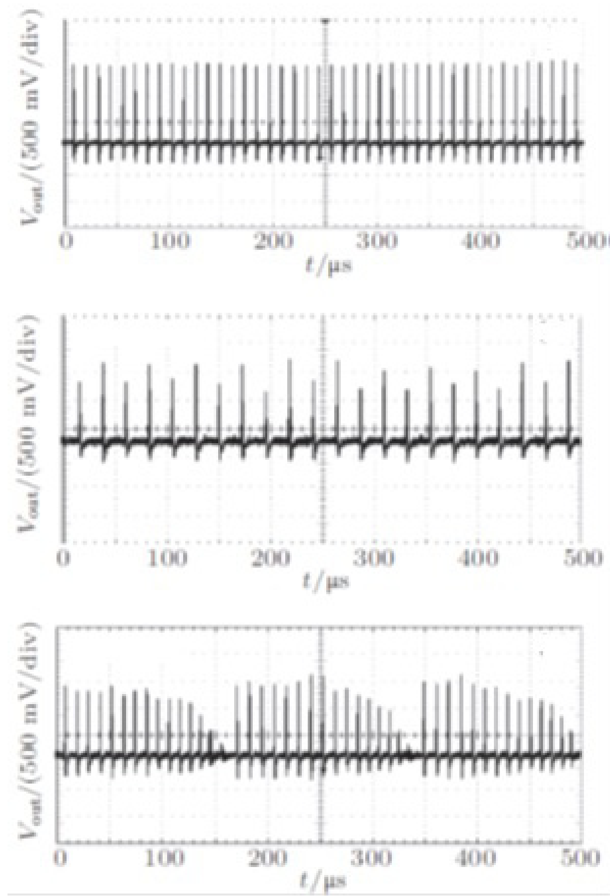

Figure 1. The temporal dependence of V_{out} upon f : Up fig. - f = 75 MHz (1-period state), Middle fig.- f = 68MHz (2-period state), Down fig.- f = 64MHz (chaos).

The theoretical examination shows that depending on the values of *f, V* the laser device is in the one-period (*f* = 75 MHz, *V* = 10V or *f* = 60MHzs, *V* = 4V), two-period (*f*= 68MHz, *V* = 10V or *f* = 60MHz, *V* = 6V), chaotic (*f* = 64MHz, *V* = 10V or f = 60MHz, *V*=10V) states.

Further we have calculated the LE values, correlation dimension, the Kaplan- York dimension, the Kolmogorov entropy and other quantities for two measured temporal series on the above descrinbed methods and algorithms. In table 1 we present the computed values of the Lyapunov's exponents LE λ_1–λ_4 in the descending order and the Kaplan- York dimension, the Kolmogorov entropy for two series of measurements.

Table 1.

Numerical parameters of the chaotic regime in the the erbium one-ring fibre laser with the control parameters: the modulation frequency f and dc bias voltage of the electro-optical modulator: □$_1$□□$_4$ are the Lyapunov exponents in descending order, d_L the Kaplan- York dimension; K – Kolmogorov entropy (our data)

Series	λ1	λ2	λ3
Exp I	0.168	0.0212	-0.223
Exp II	0.172	0.0215	-0.220
Series	λ4	d_L	Kentr
Exp I	-0.323	2.85	0.19
Exp II	-0.318	2.88	0.19

In whole application of the non-linear analysis, chaos theory and information technology methods [7-18] to studying non-linear dynamics of the erbium one-ring fibre laser (EDFL, 20.9mV strength, λ= 1550.190nm) with the control parameters: the modulation frequency *f* and dc bias voltage of the electro-optical modulator shows that there is a chaos in the erbium fiber laser device, generated via intermittency by increasing the DC bias voltage and the period-doubling bifurcation scenario by reducing the frequency modulation.

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This article has been received within 2015

UDC 541.13

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NEW NONLINEAR ANALYSIS, CHAOS THEORY AND INFORMATION TECHNOLOGY APPROACH TO STUDYING DYNAMICS OF THE QUANTUM GENERATOR AND LASER SYSTEMS

Abstract

Within new non-linear analysis, chaos theory and information technology approach it is numerically investigated chaos dynamics generation in the erbium one-ring fibre laser (EDFL, 20.9mV strength, l= 1550.190nm) with the control parameters: the modulation frequency f and dc bias voltage of the electro-optical modulator. It is shown that in depending upon f, V values there are realized 1-period f = 75MHz, V = 10V and f = 60MHz, V = 4V), 2-period (f = 68 MHz, V = 10V or f = 60MHz, V = 6V), chaotic (f = 64MHz, V = 10 V and f = 60MHz, V = 10V) regimes. There are calculated the Lyapunov's exponents, correlation, embedding, Kaplan-York dimensions, Kolmogorov entropy. Theoretically it is shown that a chaos in the erbium fiber laser device is generated via intermittency by increasing the DC bias voltage and period-doubling bifurcation by reducing the modulation frequency.

Keywords: laser system, dynamics, chaos, nonlinear analysis

УДК 541.13

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НОВЫЙ ПОДХОД НА ОСНОВЕ НЕЛИНЕЙНОГО АНАЛИЗА, ТЕОРИИ ХАОСА И ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ К ИЗУЧЕНИЮ ДИНАМИКИ КВАНТОВЫХ ГЕНЕРАТОРОВ И ЛАЗЕРНЫХ СИСТЕМ

Резюме.

На основе нового похода, включающего методы нелинейного анализа, теории хаоса и информационных технологий численно исследована динамика генерации хаоса в эрбиевом одно-кольцевом волоконном лазере (EDFL, 20.9mV, l= 1550.190nm) с управляющими параметрами: частотой модуляции *f* и постоянным напряжением смещения электрооптического модулятора. Показано, что в зависимости от *f, V* в системе реализуются одно-периодный (*f* = 75MHz, V = 10V and *f* = 60MHz, V = 4V), 2-периодный (*f* = 68 MHz, V = 10V or *f* = 60MHz, V = 6V), и хаотический (*f* = 64MHz, V = 10 V and *f* = 60MHz, V = 10V) режимы. Теоретически определены показатели Ляпунова, размерности вложения, Каплана-Йорка, энтропия Колмогорова и др. Теоретически показано, что хаос в эрбиевом волоконном лазере генерируется посредством перемежаемости при увеличения напряжения смещения постоянного тока и через бифуркации удвоения периода при уменьшения частоты модуляции.

Ключевые слова: лазерная система, динамика, хаос, нелинейный анализ

УДК 541.13

Г. П. Препелиця

НОВИЙ ПІДХІД НА ОСНОВІ НЕЛІНІЙНОГО АНАЛІЗУ, ТЕОРІЇ ХАОСУ ТА ІНФОРМАЦІЙНИХ ТЕХНОЛОГІЙ ДО ВИВЧЕННЯ ДИНАМІКИ КВАНТОВИХ ГЕНЕРАТОРІВ І ЛАЗЕРНИХ СИСТЕМ

Резюме.

На основі нового походу, що включає методи нелінійного аналізу, теорії хаосу та інформаційних технологій, чисельно досліджена динаміка генерації хаосу в ербієвому одно-кільцевому волоконному лазері (EDFL, 20.9mV, l = 1550.190nm) з керуючими параметрами: частотою модуляції *f* і постійною напругою зміщення електрооптичного модулятора. Показано, що залежно від *f, V* в системі реалізуються одно-періодний (*f* = 75MHz, V = 10V and *f* = 60MHz, V = 4V), 2-періодний (*f* = 68 MHz, V = 10V or *f* = 60MHz, V = 6V) і хаотичний (*f* = 64MHz, V = 10 V and *f* = 60MHz, V = 10V). Теоретично визначені показники Ляпунова, кореляційна розмірність, розмірності вкладення, Каплана-Йорка, ентропія Колмогорова та ін. Теоретично показано, що хаос в ербієвому волоконному лазері генерується за допомогою переміжаємості при збільшенні напруги зсуву постійного струму і скрізь біфуркації подвоєння періоду при зменшення частоти модуляції..

Ключові слова: лазерна система, динаміка, хаос, нелінійний аналіз