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## RELATIVISTIC THEORY OF SPECTRA OF PIONIC ATOMS: RADIATION TRANSITION PROBABILITIES

A new theoretical approach to the description of spectral parameters pionic atoms in the excited states with precise accounting relativistic, radiation and nuclear effects is applied to the study of radiation parameters of transitions between hyperfine structure components. As an example of the present approach presents new data on the probabilities of radiation transitions between components of the hyperfine structure transitions 5g-4f, 5f-4d in the spectrum of pionic nitrogen are presented and it is performed comparison with the corresponding theoretical data by Trassinelli-Indelicato

### 1. Introduction

Our work is devoted to the further application of earlier developed new theoretical approach [1-3] to the description of spectra and different spectral parameters, in particular, radiative transitions probabilities for pionic atoms in the excited states with precise accounting relativistic, radiation. Here problem to be solved is estimate of the hyperfine structure components transitions probabilities. As it was indicated earlier [1-3] nowadays investigation of the pionic and at whole the exotic hadronic atomic systems represents a great interest as from the viewpoint of the further development of atomic and nuclear spectral theories as creating new tools for sensing the nuclear structure and fundamental pion-nucleus strong interactions [1-15]. It is, above all, the strong pion-nucleon interaction, new information about the properties of nuclei and hadrons themselves and their interactions with the nucleus of the measured energy X-rays emitted during the transition pion spectrum of the atom. That is, optics and spectroscopy of pion atoms already in the electromagnetic sector is extremely valuable area of research that provide unique data for different areas of physics. It should be emphasized that the theory of pion

spectra of atoms are highly excited, even in the electromagnetic sector (ie short-range strong pion-N interaction

neglects little) is extremely complex and at present, despite the known progress remains very poorly developed. It is about the fundamental theoretical problems describing relativistic atoms considering nuclear, radiation effects, and a completely insufficient spectral data for pion atoms. While determining the properties of pion atoms in theory is very simple as a series of H such models and more sophisticated methods such combination chiral perturbation theory (TC), adequate quantitative description of the spectral properties of atoms in the electromagnetic pion sector (not to mention even the strong interaction sector) requires the development of High-precision approaches, which allow you to accurately describe the role of relativistic, nuclear, radiation QED (primarily polarization electron-positron vacuum, etc.). pion effects in the spectroscopy of atoms.

The most popular theoretical models are naturally (pion is the Boson with spin 0, mass:

$$m_{\pi^-} = 139.57018 \text{ MeV},$$

$$r_{\pi^-} = 0.672 \pm 0.08 \text{ fm}$$

based on the using the Klein-Gordon-Fock equation, but there are many important problems connected with accurate accounting for as pion-nuclear strong interaction effects as QED radiative corrections (firstly, the vacuum polarization effect etc.). This topic has been a subject of intensive theoretical and experimental interest (see [1-16]). The perturbation theory expansion on the physical; parameter  $\alpha Z$  is usually used to take into account the radiative QED corrections, first of all, effect of the polarization of electron-positron vacuum etc. This approximation is sufficiently correct and comprehensive in a case of the light pionic atoms, however it becomes incorrect in a case of the heavy atoms with large charge of a nucleus  $Z$ .

The more correct accounting of the QED, finite nuclear size and electron-screening effects for pionic atoms is also very serious and actual problem to be solved more consistently in comparison with available theoretical models and schemes. At last, a development of the comprehensive theory of hyperfine structure and computing radiative transitions probabilities between its components is of a great interest and importance in a modern theory of the pionic atom spectra.

## 2. Theory

The basic topics of our theoretical approach have been earlier presented [1-3], so here we are limited only by the key elements. Naturally, the relativistic dynamic of a spinless boson (pion) particle is described by the Klein-Gordon-Fock (KGF) equation. As usually, an electromagnetic interaction between a negatively charged pion and the atomic nucleus can be taken into account introducing the nuclear potential  $A_v$  in the KG equation via the minimal coupling  $p_v \rightarrow p_v - qA_v$ . The relativistic wave functions of the zeroth approximation for pionic atoms are determined from the KGF equation [1]:

$$m^2 c^2 \Psi(x) = \left\{ \frac{1}{c^2} [i\hbar \partial_t + eV_0(r)]^2 + \hbar^2 \nabla^2 \right\} \Psi(x) \quad (1)$$

where  $h$  is the Planck constant,  $c$  the velocity of the light and the scalar wavefunction  $\Psi_0(x)$  depends on the space-time coordinate  $x = (ct, r)$ .

Here it is considered a case of a central Coulomb potential ( $V_0(r), 0$ ). Then the standard stationary equation looks as:

$$\left\{ \frac{1}{c^2} [E + eV_0(r)]^2 + \hbar^2 \nabla^2 - m^2 c^2 \right\} \varphi(x) = 0 \quad (2)$$

where  $E$  is the total energy of the system (sum of the mass energy  $mc^2$  and binding energy  $\epsilon_0$ ). In principle, the central potential  $V_0$  should include the central Coulomb potential, the radiative (in particular, vacuum-polarization) potential as well as the electron-screening potential in the atomic-optical (electromagnetic) sector. Surely, the full solution of the pionic atom energy especially for the low-excited state requires an inclusion the pion-nuclear strong interaction potential. However, the main problem considered here is computing the radiative transitions probabilities between components of the hyperfine structure for sufficiently high states, when the strong pion-nuclear interaction is not important from the quantitative viewpoint. However, if a pion is on the high orbit of the atom, the strong interaction effects can not be accounted because of the negligible value.

The next step is accounting the nuclear finite size effect or the Breit-Rosenthal-Crawford-Schawlow one. In order to do it we use the widespread Gaussian model for nuclear charge distribution. The advantages of this model in comparison with usually used models such as for example an uniformly charged sphere model and others had been analysed in Ref. [1-]. Usually the Gauss model is determined as follows:

$$\rho(r|R) = \left( 4\gamma^{3/2} / \sqrt{\pi} \right) \exp(-\gamma r^2), \quad (3)$$

where  $\gamma = 4\pi / R^2$ ,  $R$  is an effective radius of a nucleus.

In order to take into account very important radiation QED effects we use the radiative potential from the Flambaum-Ginges theory [15]. It includes the standard Ueling-Serber potential and electric and magnetic form-factors plus potentials for accounting of the high order QED corrections such as:

$$\begin{aligned} \Phi_{rad}(r) = & \Phi_U(r) + \Phi_g(r) + \Phi_f(r) + \\ & + \Phi_l(r) + \frac{2}{3} \Phi_U^{high-order}(r) \end{aligned} \quad (4)$$

where

$$\Phi_U^{high-order}(r) = -\frac{2\alpha}{3\pi}\Phi(r)\frac{0.092Z^2\alpha^2}{1+(1.0 r/r_c)^4}.$$

$$\Phi_l(r) = -\frac{B(Z)}{e}Z^4\alpha^5 m^2 e^{-Z/a_B} \quad (5)$$

Here  $e$  – a proton charge and universal function  $B(Z)$  is defined by expression:  $B(Z)=0.074+0.35Z\alpha$ .

At last to take into account the electron screening effect we use the standard procedure, based on addition of the total interaction potential SCF potential of the electrons, which can be determined within the Dirac-Fock method by solution of the standard relativistic Dirac equations. It should be noted however, that contribution of these corrections is practically zeroth for the pionic nitrogen, however it can be very important in transition to many-electron as a rule heavy pionic atoms.

As we are planning to consider the radiative transitions in heavy pionic atoms in future, this block is remained in our approach.

Further in order to calculate probabilities of the radiative transitions between energy level of the pionic atoms we have used the well known relativistic energy Ivanova-Ivanov approach (look [17-19] and Refs. in [16], which is used for computing probabilities.

The expression for the energy of the hyperfine splitting (magnetic part of) the energy levels of the atom in the pion:

$$E_1^{nIF} = \frac{\mu_l \mu_N e \mu_0 \hbar c^2}{4\pi(E_0^{nl} - \langle nl|V_0(r)|nl \rangle)} \times$$

$$\times \left[ \frac{F(F+1) - I(I+1) - l(l+1)}{2I} \right] \langle nl|r^{-3}|nl \rangle \quad (6)$$

Here  $\mu_N = e\hbar/2m_p c$ ; other notations are standard. In a consistent precise theory it is important allowance for the contribution to the energy of the hyperfine splitting of the levels in the spectrum of the pion atom due to the interaction of the orbital momentum of the pion with the quadrupole moment of the atomic nucleus. The corresponding part can be presented as follows [3]:

$$\langle LIFM|W_Q|LIFM \rangle = \Delta + BC(C+1) \quad (7)$$

where

$$C = F(F+1) - L(L+1) - I(I+1), \quad (8)$$

$$B = -\frac{3}{4} \frac{e^2 Q}{I(2I-1)} \frac{(\gamma \cdot L \|\eta_2\| \gamma \cdot L)}{\sqrt{L(L+1)(2L-1)(2L+1)(2L+3)}} \quad (9)$$

$$\Delta = \frac{e^2 Q(I+1)}{(2I-1)} \frac{(\gamma \cdot L \|\mu_2\| \gamma \cdot L)L(L+1)}{\sqrt{L(L+1)(2L-1)(2L+1)(2L+3)}} \quad (10)$$

Here  $L$  – is orbital moment of pion,  $F$  is a total moment of an atom.

### 3. Results and conclusions

As example of application of the presented approach, in table 1 we present the data on radiative transition probabilities (in  $s^{-1}$ ) for hyperfine transitions 5g-4f in the spectrum of the pion nitrogen: Th1- data by Trassinelli-Indelicato; Th2- our data. In theory by Trassinelli-Indelicato (look, for example, [4])

it has been used the standard atomic spectroscopy amplitude scheme when the transitions energies and probabilities are calculated in the known degree separately. At the same time this computing within the relativistic energy approach is performed more correctly and self-consistently (look details in [16] and multiple references therein).

Table 1.  
**Radiative transition probabilities (in  $s^{-1}$ ) for hyperfine transitions 5g-4f in the spectrum of the pion nitrogen: Th1- data by Trassinelli-Indelicato; Th2- our data**

F-F'	T.I : P (5g-4f)	T.II : P (5g-4f)
5-4	$7.13 \times 10^{13}$	$7.04 \times 10^{13}$
4-3	$5.47 \times 10^{13}$	$5.41 \times 10^{13}$
4-4	$5.27 \times 10^{13}$	$5.23 \times 10^{13}$
3-2	$4.17 \times 10^{13}$	$4.12 \times 10^{13}$
3-3	$0.36 \times 10^{13}$	$0.34 \times 10^{13}$
3-4	$0.01 \times 10^{13}$	$0.009 \times 10^{13}$

In table 2 we present our data for radiative transition probabilities (in  $s^{-1}$ ) for hyperfine transitions 5f-4d in the spectrum of the pion nitrogen: our data

Table 2.

**Radiative transition probabilities (in  $s^{-1}$ ) for hyperfine transitions 5f-4d in the spectrum of the pion nitrogen: our data**

F-F'	Our data (5f-4d)
4-3	$4.57 \times 10^{13}$
3-2	$3.16 \times 10^{13}$
3-3	$2.98 \times 10^{13}$
2-1	$2.13 \times 10^{13}$
2-2	$2.25 \times 10^{13}$
2-3	$0.01 \times 10^{13}$

In whole, the computed radiative transition probabilities values for considered transitions between hyperfine structure components in the spectrum of the pion within theory by Trassinelli-Indelicato and ours demonstrate physically reasonable agreement. , however our values are a little lower. This fact can be explained by difference in the computing schemes and different level of accounting for nuclear finite size, QED and other effects (look details [1-3,20,21]). In any case the data obtained can be considered as sufficiently accurate ones and used in the corresponding applications, indicated in the introduction. There is a great interest the detailed studying the radiative transitions parameters for the heavy pionic toms especially in the Rydberg states. This topic will be considered by us in the next publications.

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### **Abstract.**

A new theoretical approach to the description of spectral parameters pionic atoms in the excited states with precise accounting relativistic, radiation and nuclear effects is applied to the study of radiation parameters of transitions between hyperfine structure components. As an example of the present approach presents new data on the probabilities of radiation transitions between components of the hyperfine structure transitions 5g-4f, 5f-4d in the spectrum of pionic nitrogen are presented and it is performed comparison with the corresponding theoretical data by Trassinelli-Indelicato.

**Keywords:** relativistic theory, hyperfine structure, pionic atom, radiative transitions

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## **РЕЛЯТИВИСТСКАЯ ТЕОРИЯ СПЕКТРОВ ПИОННЫХ АТОМОВ С УЧЕТОМ ЯДЕРНЫХ И РАДИАЦИОННЫХ ПОПРАВОК: ВЕРОЯТНОСТИ РАДИАЦИОННЫХ ПЕРЕХОДОВ**

### **Резюме.**

Новый теоретический подход к описанию спектральных параметров пионных атомов в возбужденном состоянии с учетом релятивистских, радиационных эффектов применен к изучению параметров радиационных переходов между компонентами сверхтонкой структуры. В качестве примера применения представленного подхода, представлены новые данные о вероятности радиационных переходов между компонентами сверхтонкой структуры переходов  $5g-4f$ ,  $5f-4d$  в спектре пионного азота и проведено сравнение с соответствующими теоретическими данными Trassinelli-Indelicato.

**Ключевые слова:** релятивистская теория, сверхтонкая структура, пионный атом, радиационные переходы

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## **РЕЛЯТИВІСТСЬКА ТЕОРІЯ СПЕКТРІВ ПІОННИХ АТОМІВ З УРАХУВАННЯМ ЯДЕРНИХ ТА РАДІАЦІЙНИХ ПОПРАВОК: ЙМОВІРНІСТІ РАДІАЦІЙНИХ ПЕРЕХОДІВ**

### **Резюме.**

Новий теоретичний підхід до опису спектральних параметрів піонних атомів у збудженому стані з урахуванням релятивістських, радіаційних ефектів на основі рівняння Клейна-Гордона-Фока застосовано до вивчення параметрів радіаційних переходів між компонентами надтонкої структури. Як приклад застосування представленого підходу, представлені нові дані про ймовірностей радіаційних переходів між компонентами надтонкої структури переходів  $5g-4f$ ,  $5f-4d$  в спектрі піонного азоту і проведено порівняння з відповідними теоретичними даними Trassinelli-Indelicato.

**Ключові слова:** релятивістська теорія, надтонка структура, піоний атом, радіаційні переходи