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NONLINEAR DYNAMICS OF RELATIVISTIC BACKWARD-WAVE TUBE IN SELF-MODULATION AND CHAOTIC REGIME

It has been performed quantitative modelling, analysis, forecasting dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects ($\gamma_0 = 1.5-6.0$), dissipation (factor D), a presence of space charge etc. There are computed the temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N , bifurcation parameter proportional to (\sim current I) Pirse one J and relativistic factor γ_0 . The computed temporal dependence of the field amplitude (power) F_{max} in a good agreement with theoretical estimates and experimental data by Ginzburg et al (IAP, Nizhny Novgorod) with using the pulsed accelerator "Saturn". The analysis techniques including multi-fractal approach, methods of correlation integral, false nearest neighbour, Lyapunov exponent's, surrogate data, is applied analysis of numerical parameters of chaotic dynamics of RBWT. There are computed the dynamic and topological invariants of the RBWT dynamics in auto-modulation(AUM)/chaotic regimes, correlation dimensions values (3.1; 6.4), embedding, Kaplan-York dimensions, Lyapunov's exponents (+,+) Kolmogorov entropy. There are constructed the bifurcation diagrams with definition of the dynamics self-modulation/chaotic areas in planes, namely, " $J-\gamma_0$ ", " $D-J$ ".

1. Introduction

As it is well known in the modern electronics, photoelectronics etc there are many physical systems (the backward-wave tubes, multielement semiconductors and gas lasers, different radio-technical devices etc), which can manifest the elements of chaos and hyperchaos in their dynamics (c.f.[1-32]). The key aspect of studying the dynamics of these systems is analysis of the dynamical characteristics. Chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. The past decade has witnessed a large number of studies employing the ideas gained from the science of chaos to characterize, model, and predict the dynamics of various systems phenomena (c.f.[1-16]). The outcomes of such studies are very encouraging, as they not only revealed that the dynamics of the apparently

irregular phenomena could be understood from a chaotic deterministic point of view but also reported very good predictions using such an approach for different systems.

The backward-wave tube is an electronic device for generating electromagnetic vibrations of the superhigh frequencies range. In refs.[3-16] there have been presented the temporal dependences of the output signal amplitude, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and for developed chaos at large values of the dimensionless length parameter. The authors of [3-16] solved the different versions of system of equations of nonstationary nonlinear theory for the O type backward-wave tubes with and without account of the spatial charge, without energy losses etc. It has been shown that the finite-dimension strange attractor is responsible for chaotic regimes in the backward-wave tube.

In our work it has been performed quantitative modelling, analysis, forecasting dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects ($g_0 = 1.5-6.0$), dissipation, a presence of space charge etc. There are computed the temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N , bifurcation parameter proportional to (\sim current I) Pirsone and relativistic factor g_0 . There is computed a temporal dependence of the field amplitude (power) F_{\max} in a good agreement with theoretical estimates and experimental data by Ginzburg et al (IAP, Nizhny Novgorod) with using the pulsed accelerator "Saturn".

2. Method

As the key ideas of our technique for nonlinear analysis of chaotic systems have been in details presented in refs. [17-32], here we are limited only by brief representation. The first important step is a choice of the model of the RBWT dynamics. We use the standard non-stationary theory [3-6], however, despite the cited papers we take into account a number of effects, namely, influence of space charge, dissipation, the waves reflections at the ends of the system and others [12,13]. Usually relativistic dynamics is described system of equations for unidimensional relativistic electron phase $\theta(\zeta, \tau, \theta_0)$ (which moves in the interaction space with phase q_0 ($q_0 \hat{I}[0; 2\pi]$) and has a coordinate z at time moment t) and field $E(x, t) = \text{Re}[\varepsilon(x, t) \exp[i\omega_0 t - i\beta_0 x]]$ unidimensional complex amplitude $F(\zeta, \tau) = \tilde{E} / (2\beta_0 U C^2)$ as follows [240, 249]:

$$\frac{\partial^2 \theta}{\partial \zeta^2} = -L^2 \gamma_0^3 \left[\left(1 + \frac{1}{2\pi N} \frac{\partial \theta}{\partial \zeta} \right)^2 - \beta_0^2 \right]^{3/2} \times \text{Re}[F \exp(i\theta)], \quad (1)$$

$$\frac{\partial F}{\partial \tau} - \frac{\partial F}{\partial \zeta} = -L \tilde{I}, \quad \tilde{I} = -\frac{1}{\pi} \int_0^{2\pi} e^{-i\theta} d\theta_0$$

with the boundary and initial conditions:

$$\theta|_{\zeta=0} = \theta_0, \quad \frac{\partial \theta}{\partial \zeta}|_{\zeta=0} = 0$$

$$F|_{\zeta=1} = 0, \quad F|_{\tau=0} = F^0(\zeta)$$

It is important to note that the system studied has a few controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N , bifurcation parameter proportional to (\sim current I) Pirsone $L = 2\pi CN / \gamma_0$ (here C is the known Piers parameter, $C = \sqrt[3]{I_0 K_0 / (4U)}$, and I_0 is a constant beam current component, U - accelerating voltage, K - resistance of coupling of the slowing down system) and relativistic factor $\gamma_0 = (1 - \beta_0^2)^{-1/2}$.

Since processes resulting in the chaotic behaviour are fundamentally multivariate, it is necessary to reconstruct phase space using as well as possible information contained in the dynamical parameter $s(n)$, where n the number of the measurements. Such a reconstruction results in a certain set of d -dimensional vectors $y(n)$ replacing the scalar measurements. Packard et al. [19] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n+t)$, where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector in d dimensions,

$$y(n) = [s(n), s(n+t), s(n+2t), \dots, s(n+(d-1)t)], \quad (1)$$

the required coordinates are provided. In a nonlinear system, the $s(n+jt)$ are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension d is called the embedding dimension, d_E . According to Mañé and Takens [24,25], any time lag will be acceptable is not terribly useful for extracting physics from data. If t is chosen too small, then the coordinates $s(n+jt)$ and $s(n+(j+1)t)$ are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if t is too large, then $s(n+jt)$ and $s(n+(j+1)t)$ are completely independent of each other in a statistical sense. Also, if t is too small or too large, then the correlation

dimension of attractor can be under- or overestimated respectively. The autocorrelation function and average mutual information can be applied here. The first approach is to compute the linear autocorrelation function:

$$C_L(\delta) = \frac{\frac{1}{N} \sum_{m=1}^N [s(m+\delta) - \bar{s}] [s(m) - \bar{s}]}{\frac{1}{N} \sum_{m=1}^N [s(m) - \bar{s}]^2}, \quad (2)$$

$$\bar{s} = \frac{1}{N} \sum_{m=1}^N s(m)$$

and to look for that time lag where $C_L(d)$ first passes through zero (see [18]). This gives a good hint of choice for t at that $s(n+jt)$ and $s(n+(j+1)t)$ are linearly independent. A time series under consideration have an n -dimensional Gaussian distribution, these statistics are theoretically equivalent as it is shown by Paluš (see [15]). The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible non-linear nature of process resulting in the vibrations amplitude level variations can be concluded.

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding dimension, d_E , must be greater, or at least equal, than a dimension of attractor, d_A , i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E , e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [5,6,18]. First, many of computations for extracting interesting properties from the data require searches and other operations in R^d whose computational cost rises exponentially with d . Second, but more significant from the physical point of view, in the presence of noise or other high dimensional contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal. In too large an embedding space one is unnecessarily spending time working around aspects of a bad representation of the

observations which are solely filled with noise. It is therefore necessary to determine the dimension d_A .

There are several standard approaches to reconstruct the attractor dimension (see, e.g., [3-6,15]). The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [10] is the most commonly used approach. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r \rightarrow 0 \\ N \rightarrow \infty}} \frac{\log C(r)}{\log r}, \quad (3)$$

where d is correlation exponent that can be determined as the slope of line in the coordinates $\log C(r)$ versus $\log r$ by a least-squares fit of a straight line over a certain range of r , called the scaling region. If the correlation exponent attains saturation with an increase in the embedding dimension, the system is generally considered to exhibit chaotic dynamics. The saturation value of correlation exponent is defined as the correlation dimension (d_2) of attractor.

The Lyapunov exponents are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local Lyapunov exponents.

A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of Lyapunov exponents is considered a measure of the effect of perturbing the initial conditions of a dynamical system. Since the Lyapunov exponents are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of Lyapunov exponents, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, K , measures the average rate at which information about

the state is lost with time. An estimate of this measure is the sum of the positive Lyapunov exponents. The inverse of the Kolmogorov entropy is equal to the average predictability. There are several approaches to computing the Lyapunov exponents (see, e.g., [5,6,18]). One of them [18] is in computing the whole spectrum and based on the Jacobin matrix of the system function [14].

3. Results

As input, the following parameters were taken: the energy of electrons - 150keV, starting current of 7A composed impedance connection 0,5W , length of interaction space - 0,623m, the average radius waveguides - 1,38sm period corrugating - 1,73sm radius of the electron beam - 0,67sm. The dynamic model (2.6) has been implemented in two ways considering the effects of space charge and without and with (unlike in [5]) the effect of slowing the loss of energy in the system (at the ends of reflection and some other factors discussed more etc.). As bifurcation parameter actually is $J = \mathcal{E} |Z| (2\beta_0^2 m^{-2})$, where Z - resistance connection, I - beam current, $\beta_0 = v_0 / c$, v_0 - the initial velocity of the electrons, the parameter space charge $Q = Ieg (m \omega^2 b)$, transverse wave number $g = \omega (c\beta_0\gamma_0)$, k-harmonic and space charge density $q_k = (1/\pi) \int_0^{2\pi} e^{-k\theta} d\theta_0$, coefficient of reduction space charge $f_k^0 = 0.55$. To factor in the expression for the normalized dissipation parameter has been fixed $D = 8\text{Db}$. In figure 1 we list the relevant theoretical simulation test results in non-stationary processes RBWT at injection currents: (a) - 55A, (b) - 90A, (c) - 120A.

At current 7A it is set stationary mode that with increasing value of current strength transited to the periodic automodulation ($I = 30\text{A}$, on our data, the period of $T_a = 7,3\text{ns}$; experimental value [14b]: 8ns), and then when $I = 55\text{A}$ it is realized the chaotic auto-modulation mode (fig 1a). By increasing the amount of current to 75A there is the quasi-periodical auto-modulation (period 13.8 ns) and, finally, when the current value is more than 100A it's realized essentially chaotic regime. Note that reset of the quasi-periodic auto-modulation mode can be explained by an effect of space charge.

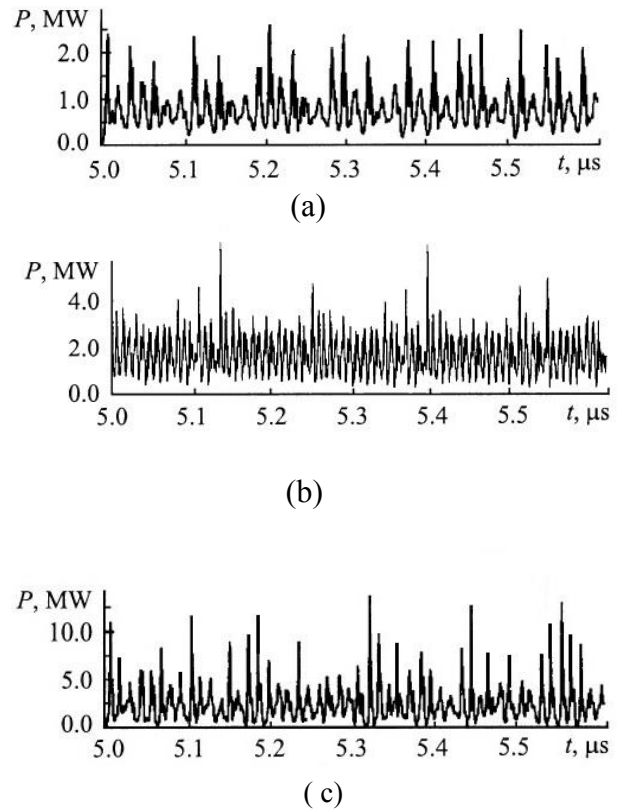


Figure 1. Theoretical results for the temporal dependence of power of the RBWT at the injection currents: (a)-55A, (b)-90A, (c) - 120A.

The similar theoretical estimates (however without the dissipation effect) and experiment results data have been obtained by Ginsburg et al. [5b]. Let us note that all results are in a physically reasonable agreement with each other.

Fig. 2 (a) shows the results of our computing the autocorrelation function, and Fig. 2b) - the average mutual information.

In fig.3 there is listed the relationship between the correlation exponent and embedding dimension of the temporal series (line 1), the mean values of variables replacement (line 2) and the implementation of one replacement (line 3). Columns errors indicate minimum values exponential correlation among all variables substituted. In Fig. 4 we presents data of estimating the embedding dimension based on the algorithm of false nearest neighbours for points of the original data series (line 1), the mean values of surrogate data (2), and one surrogate realization (3).

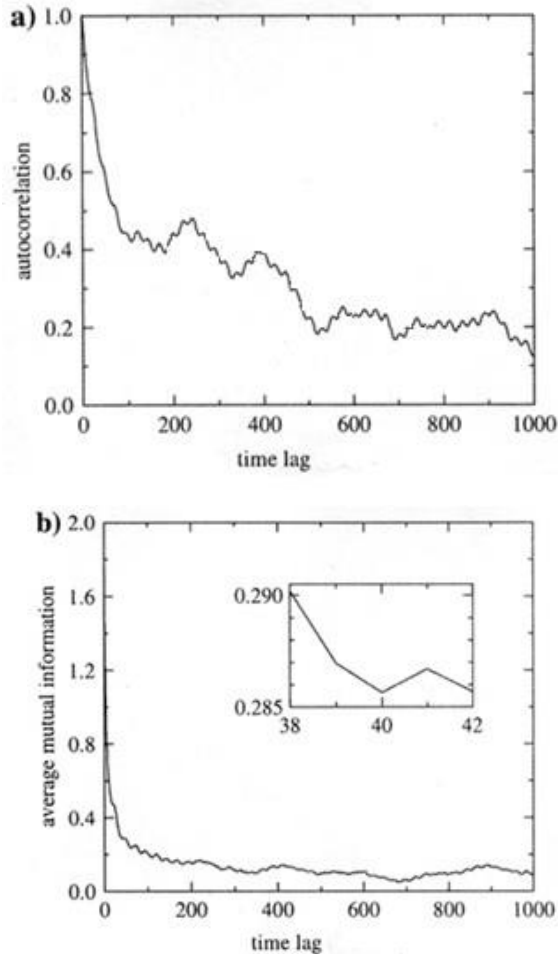


Figure 2. The autocorrelation function, (a) and the average mutual information (b).

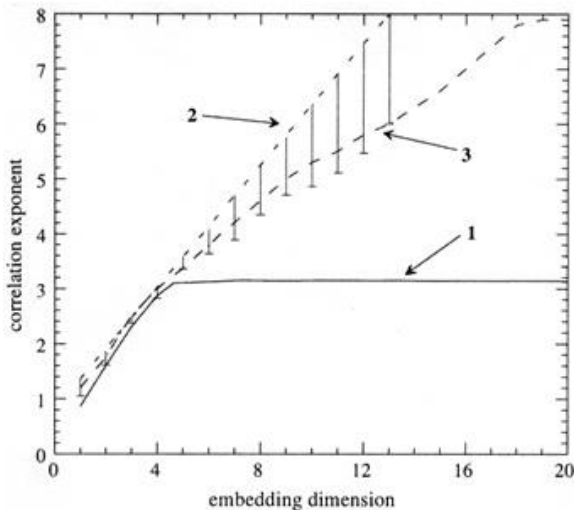


Figure 3. The relationship between the correlation exponent and embedding dimension of the temporal series (line 1), the mean values of variables replacement (line 2) and the implementation of one replacement (line 3).

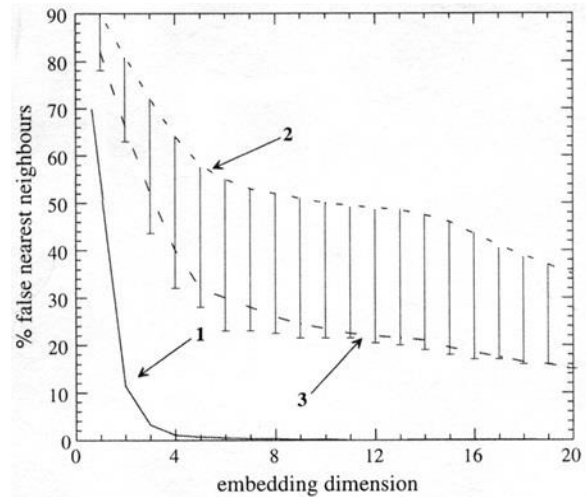


Fig. 4 we presents data of estimating the embedding dimension based on the algorithm of false nearest neighbours for points of the original data series (line 1), the mean values of surrogate data (2), and one surrogate realization (3).

Next in the table 1 we list our data on . the correlation dimension d_2 , embedding dimension, determined on the basis of false nearest neighbours algorithm (d_N) with percentage of false neighbours (%). calculated for different values of lag t according to the analysis of two series fig1a (I - chaos) and fig.1c (II - hyperchaos).

Table 1
Correlation dimension d_2 , embedding dimension, determined on the basis of false nearest neighbours algorithm (d_N) with percentage of false neighbours (%) calculated for different values of lag t

Chaos (I)			Hyperchaos (II)		
τ	d_2	(d_N)	τ	d_2	(d_N)
60	3.6	5 (5.5)	67	7.2	10 (12)
6	3.1	4 (1.1)	10	6.3	8 (2.1)
8	3.1	4 (1.1)	12	6.3	8 (2.1)

In Table 2 we list our computing data on the Lyapunov exponents (LE), the dimension of the Kaplan-York attractor, the Kolmogorov entropy K_{entr} . For studied series there are the positive and negative LE values. The resulting dimension Kaplan York in both cases are very similar to the correlation dimension (calculated by the algorithm by Grassberger-Procaccia).

Table 1
The Lyapunov exponents (LE), the dimension of the Kaplan-York attractor, the Kolmogorov entropy K_{entr} . (our data)

Chaos	λ_1	λ_2	λ_3	λ_4	K
(I)	0.261	0.0001	-0.0004	-0.528	0.26
(II)	0.514	0.228	0.0000	-0.0002	0.74

Further, in Fig.5 we present the firstly obtained original (continuous line) and predicted (dotted line) dependences of power in the chaos mode (I): (a) - without energy loss effect, (b) - taking into account the effect of loss. In order to estimate reliability (success) of prediction model [13-15] we have computed the correlation coefficient (r) between actual and

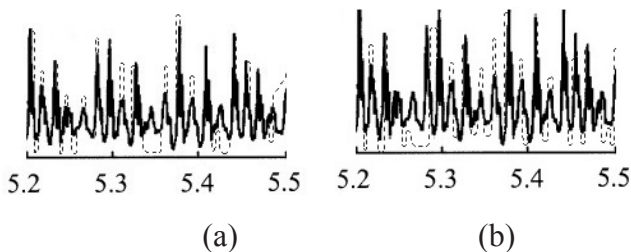


Figure 5. Original (continuous line) and predicted (dotted line) dependences of power in the chaos mode (I): (a) - without energy loss effect, (b) - taking into account the effect of loss

prognostic rows ranked to a number of the neighbours (NN). In this case, the mean forecast error was ($\sigma = 1.9$) for time series (chaos mode). In addition, usually to account for a forecast error one should take into account the noise level in the studied time series. For this purpose the methodology by Hu et al (see [13]) was used.

Importantly, the above-described physical mechanism of changing different modes in the RBWT dynamics due to an increasing a current value and the bifurcation parameter J corresponds to certain value relativistic factor, namely $\gamma_0 = 1,3$.

More important is the analysis of the RBWT nonlinear dynamics in the plane «relativistic factor – bifurcation parameter.» Actually in this context a three-parametric relativistic nonlinear dynamics is fundamentally different from processes in non-relativistic BWT dynamics. In fig.6 we list a chart that shows the quantitative limits of auto-modulation (line I) in the plane of parameters: bifurcation parameter J - relativistic factor γ_0 . Note that the second line (line II) limits the area where there is a twist particles and used theoretical model works. A characteristic feature of the chart is the presence of so-called effect of «beak», which is based on relativistic factor goes far deeper auto-modulation area. Firstly this effect was predicted in [3-6]. In essentially relativistic limit (see. Fig. 7) the frequency of auto-modulation falls by about half. Obviously, that all of the above characteristics is much more complicated compared to the dynamics of non-relativistic dynamics.

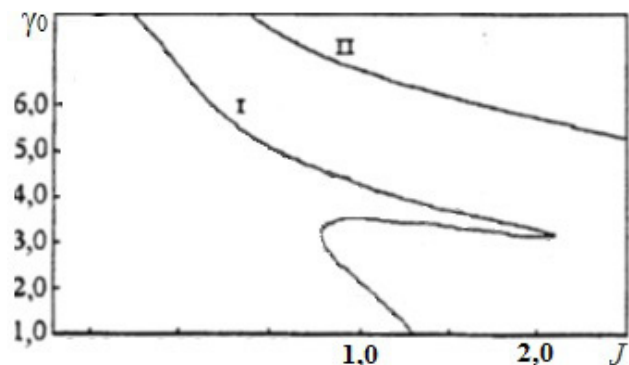


Figure 6. The limits of auto-modulation (line I) on the plane of parameters: “bifurcation parameter - relativistic factor”

So, we believe that a chaos in the RBWT dynamics should be called by relativistic chaos phenomenon.

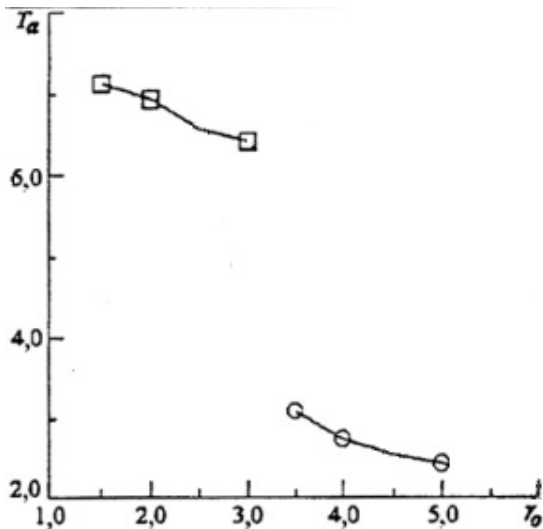


Figure 7. The dependence of the frequency of auto-modulation upon relativistic factor

Conclusions

In this work we have performed quantitative modelling, analysis, forecasting dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects ($\gamma_0 = 1.5-6.0$), dissipation, a presence of space charge, reflection of waves at the end of deceleration system etc. There are computed the temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N , bifurcation parameter proportional to (\sim current I) Pirse one $L(J)$: 2.7-3.9 and relativistic factor $g_0 = 1.5-6.0$). There are computed the dynamic and topological invariants of the RBWT dynamics in auto-modulation/chaotic regimes, correlation dimensions values (3.1; 6.4), embedding, Kaplan-York dimensions, Lyapunov's exponents (LE:+,+) Kolmogorov entropy. There are firstly constructed the bifurcation diagrams with definition of the dynamics self-modulation/chaotic areas in planes: « $J-\gamma_0$ », « $D-J$ ». It is shown that for moderately small $\gamma_0 \sim 1.3$ transition to chaos is realized through a sequence of the period doubling bifurcations, but with the growth of the g_0 dynamics significantly complicates with interchange of quasi-harmonical/ chaotic regimes (incl. discovery of a "beak" effect on the chart, sharp fall of automodulation period at $\gamma_0 \sim 4$),

emergence of highly-d chaotic attractor, which evolves at a much complicated scenario. Firstly on basis of chaos-cybernetic approach with a new wavelet-expansion predicted paths algorithm it is realized forecasting the temporal evolution of chaotic dynamics for RBWT at different values of J , g_0 taking into account the effects of relativity, influence of space charge, dissipation and shown that in a case of low-attractor dynamics (chaotic auto-modulation) the predicted series well rebuilt the empirical data (correlation coefficient between predicted and real rows ranked among the neighbours number ~ 0.97), which is the first indication of the possibility of a new quantitative evolution prediction direction in studying relativistic microwave electronics devices.

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NON-LINEAR DYNAMICS OF RELATIVISTIC BACKWARD-WAVE TUBE IN SELF-MODULATION AND CHAOTIC REGIME

Abstract.

It has been performed quantitative modelling, analysis, forecasting dynamics relativistic backward-wave tube (RBWT) with accounting relativistic effects ($g_0 = 1.5-6.0$), dissipation (factor D), a presence of space charge etc. There are computed the temporal dependences of the normalized field amplitudes (power) in a wide range of variation of the controlling parameters which are characteristic for distributed relativistic electron-waved self-vibrational systems: electric length of an interaction space N , bifurcation parameter proportional to (\sim current I) Pirse one J and relativistic factor g_0 . The computed temporal dependence of the field amplitude (power) F_{\max} in a good agreement with theoretical estimates and experimental data by Ginzburg etal (IAP, Nizhny Novgorod) with using the pulsed accelerator “Saturn”. The analysis techniques including multi-fractal approach, methods of correlation integral, false nearest neighbour, Lyapunov exponent’s, surrogate data, is applied analysis of numerical parameters of chaotic dynamics of RBWT. There are computed the dynamic and topological invariants of the RBWT dynamics in auto-modulation(AUM)/chaotic regimes, correlation dimensions values (3.1; 6.4), embedding, Kaplan-York dimensions, Lyapunov’s exponents (+,+) Kolmogorov entropy. There are constructed the bifurcation diagrams with definition of the dynamics self-modulation/chaotic areas in planes, namely, “ $J-g_0$ », « $D-J$ ».

Key words: relativistic backward-wave tube, chaos, non-linear methods

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НЕЛИНЕЙНАЯ ДИНАМИКА РЕЛЯТИВИСТСКОЙ ЛАМПЫ ОБРАТНОЙ ВОЛНЫ В АВТОМОДУЛЯЦИОННОМ И ХАОТИЧЕСКОМ РЕЖИМАХ

Резюме.

Приведены результаты моделирования, анализа и прогноза динамики процессов в релятивистской лампе обратной волны (РЛОВ) с учета релятивистских эффектов ($g_0 = 1,5-6,0$), диссипации (фактор D), наличия пространственного заряда и т.д. Вычислены временные зависимости нормированной амплитуды поля (мощности) в широком диапазоне изменения управляющих параметров, которые характерны для распределенных релятивистских электронно-волновых автоколебательных систем: электрическая длина пространства взаимодействия N , бифуркационный параметр, пропорциональный силе тока, J и релятивистский фактор g_0 . Вычисленная зависимость амплитуды поля (мощности) F_{max} находится в хорошем согласии с теоретическими оценками и данными эксперимента Гинзбурга и др. (ИПФ, Нижний Новгород) с использованием импульсного ускорителя «Сатурн». Техника нелинейного анализа, которая включает мультифрактальный подход, методы корреляционных интегралов, ложных ближайших соседей, экспонент Ляпунова, суррогатных данных, использованная для анализа численных параметров хаотических автоколебательных режимов в РЛОВ. Рассчитаны динамические и топологические инварианты динамики РЛОВ в автомодуляционном и хаотическом режимах, корреляционная размерность, размерности вложения (3.1; 6.4), Каплан-Йорка, показатели Ляпунова (+, +), энтропия Колмогорова и построены бифуркационные диаграммы с определением областей автомодуляции и хаоса, в частности, « $J-g_0$ », « $D-J$ ».

Ключевые слова: релятивистская лампы обратной волны, хаос, нелинейные методы

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НЕЛІНІЙНА ДИНАМІКА РЕЛЯТИВІСТСЬКОЇ ЛАМПИ ЗВЕРНЕНОЇ ХВИЛІ В АВТОМОДУЛЯЦІЙНОМУ ТА ХАОТИЧНОМУ РЕЖИМАХ

Резюме.

Наведені результати моделювання, аналізу і прогнозу динаміки процесів в релятивістській лампі зворотної хвилі (РЛЗХ) з урахуванням релятивістських ефектів ($g_0 = 1,5-6,0$), дисипації (фактор D), наявності просторового заряду і т.і. Обчислені часові залежності нормованої амплітуди поля (потужності) в широкому діапазоні зміни керуючих параметрів, які характерні для розподілених релятивістських електронно-хвильових автоколивальних систем: електрична довжина простору взаємодії N , біфуркаційний параметр, пропорційний силі струму, J і релятивістський фактор g_0 . Обчислена залежність амплітуди поля (потужності) F_{max} знаходиться в хорошому злагоді з теоретичними оцінками і даними експерименту Гінзбурга та ін. (ІПФ,

Нижній Новгород) з використанням імпульсного прискорювача «Сатурн». Техніка нелінійного аналізу, яка включає мультифрактальний підхід, методи кореляційних інтегралів, хибних найближчих сусідів, експонент Ляпунова, сурогатних даних, використана для аналізу чисельних параметрів хаотичних автоколивальних режимів у РЛЗХ. Розраховані динамічні та топологічні інваріанти динаміки РЛОВ в автомодуляційному і хаотичному режимах, кореляційна розмірність, розмірності вкладення (3.1; 6.4), Каплан-Йорка, показники Ляпунова (+, +), ентропія Колмогорова і побудовані біфуркаційні діаграми з визначенням областей автомодуляції і хаосу, зокрема, «J-g₀», «D-J».

Ключові слова: релятивістська лампи зворотної хвилі, хаос, нелінійні методи