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NONLINEAR DYNAMICS OF QUANTUM AND LASER SYSTEMS WITH ELEMENTS OF A CHAOS

Nonlinear chaotic dynamics of the quantum and laser systems is studied with using advanced techniques such as a wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, the Lyapunov exponent's (LE) analysis, and surrogate data method. The detailed analysis of the oscillations in a grid of two autogenerators and single-mode laser with the nonlinear absorption cell shows that the systems exhibit a nonlinear behaviour with elements of a low-dimensional chaos.

1. Introduction

Every science purposes predicting a future state of system under consideration. Consequently, the main problem of science can be defined as: "Is it possible to predict a future behaviour of process using its past states?" Conventional approach applied to resolve this problem consists in building an explanatory model using an initial data and parameterizing sources and interactions between process properties. Unfortunately, that kind of approach is realized with difficulties, and its outcomes are insufficiently correct; moreover, sources and/or interactions of process cannot always be exactly defined. According to modern theory of prediction, time series is considered as random realization, when the randomness is caused by a complicated motion with many independent degrees of freedom. Chaos is alternative of randomness and occurs in very simple deterministic systems. Although chaos theory places fundamental limitations for long-rage prediction (see e.g. [1-9]), it can be used for short-range prediction since ex facte random data can contain simple deterministic relationships with only a few degrees of freedom. The systematic study of chaos is of recent date, originating in the 1960s. One important reason for this is that linear techniques, so long dominant within applied mathematics and the natural sciences, are inadequate when considering chaotic phenomena since the amazingly irregular behaviour of some non-linear deterministic systems was not appreciated and when such behaviour was manifest in observations, it was typically explained as stochastic. Starting from the meteorologist Edward Lorenz, who observed extreme sensitivity to changes to initial conditions of a simple non-linear model simulating atmospheric convection (Lorenz, 1963), the experimental approach relies heavily on the computational study of chaotic systems and includes methods for investigating potential chaotic behaviour in observational time series (see e.g. [1-6]). During the last two decades, many studies in various fields of physics, chemistry, biology, geosciences etc have appeared, in which chaos theory was applied to a great number of dynamical systems, including those are originated from nature. Chaos theory establishes that apparently complex irregular behaviour could be the outcome of a simple deterministic system with a few dominant nonlinear interdependent variables. The past decade has witnessed a large number of studies employing the ideas gained from the science of chaos to characterize, model, and predict the dynamics of various systems phenomena (see e.g. [1-13]).

The outcomes of such studies are very encouraging, as they not only revealed that the dynamics of the apparently irregular phenomena could be understood from a chaotic deterministic point of view but also reported very good predictions using such an approach for different systems.

In a case of quantum systems, using of chaos constructions may seem self-contradictory in many respects (see e.g. [6,7]). To begin with, it associates chaos, a purely classical notion, with quantum physics. Furthermore it implies that this association, which as we will see refers traditionally to the study of low-D non-interacting quantum systems, will be considered in the context of many-body physics [6]. In any case quantum chaos now mainly refers to the study of the consequences, for a quantum system, of the more or less chaotic nature of the dynamics of its classical analogue. It has followed two main avenues. The first one is based on semiclassical techniques - specifically the use of semiclassical Green's functions in the spirit of Gutzwiller's trace formulae, hich provides a link between a quantum system and its $h \to 0$ limit, the second is associated with the Bohigas-Giannoni-Schmit conjecture or related approaches Peres, which states that the spectral fluctuations of classically chaotic systems can be described using the proper ensembles of random matrices [6]. Some of the beauty of quantum chaos is that it has developed a set of tools which have found applications in a large variety of different physical contexts, ranging from atomic, molecular and nuclear physics (see e.g. [1-7]). In a modern quantum electronics and laser physics etc there are many systems and devices (such as multi-element semiconductors and gas lasers etc), dynamics of which can exhibit chaotic behaviour. These systems can be considered in the first approximation as a grid of autogenerators (quantum generators), coupled by different way [2,14,15]. In this chapter we will study a non-linear chaotic dynamics of some quantum generator and laser systems with using advanced generalized techniques such as the non-linear analysis methods to dynamics, such as the wavelet analysis, multifractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's (LE) analysis, and surrogate data method etc (see details in Refs. [8-17]).

2. Methods of a chaos theory in studying dynamics of the complex systems

2.1 Introducing remarks

Let us formally consider scalar measurements $s(n) = s(t_0 + n\mathrm{D}t) = s(n)$, where t_0 is the start time, Dt is the time step, and is n the number of the measurements. In a general case, $s(n)$ is any time series, particularly the amplitude level. Since processes resulting in the chaotic behaviour are fundamentally multivariate, it is necessary to reconstruct phase space using as well as possible information contained in the $s(n)$.

Such a reconstruction results in a certain set of d-dimensional vectors $\mathbf{y}(n)$ replacing the scalar measurements. Packard et al. [18] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n + \mathrm{t})$, where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector in d dimensions,

$$\mathbf{y}(n) = [s(n), s(n+\tau), s(n+2\tau, \ldots, s(n+(d\text{-}1)\tau], \quad (1)$$

the required coordinates are provided. In a nonlinear system, the $s(n + j\mathrm{t})$ are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension d is called the embedding dimension, d_E. Example of the Lorenz attractor given by Abarbanel et al. [1,19] is a good choice to illustrate the efficiency of the method.

2.2 Choosing time lag

According to Mañé and Takens [20,21], any time lag will be acceptable is not terribly useful for extracting physics from data. If t is chosen too small, then the coordinates s(n + jt) and s(n + (j + 1)t) are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if t is too large, then s(n + jt) and s(n + (j + 1)t) are completely independent of each other in a statistical sense. Also, if t is too small or too large, then the correlation dimension of attractor can be under- or overestimated respectively [7]. It is therefore necessary to choose some intermediate (and more appropriate) position between above cases. First approach is to compute the linear autocorrelation function

$$C_L(\delta) = \frac{\frac{1}{N}\sum_{m=1}^{N}[s(m+\delta)-\bar{s}][s(m)-\bar{s}]}{\frac{1}{N}\sum_{m=1}^{N}[s(m)-\bar{s}]^2}, \quad (2)$$

where

$$\bar{s} = \frac{1}{N}\sum_{m=1}^{N} s(m)$$

and to look for that time lag where C_L(d) first passes through zero. This gives a good hint of choice for t at that $s(n + j$t) and $s(n + (j + 1)$t) are linearly independent. However, a linear independence of two variables does not mean that these variables are nonlinearly independent since a nonlinear relationship can differs from linear one. It is therefore preferably to utilize approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows [5,7,13]. Let there are two systems, A and B, with measurements a_i and b_k. The amount one learns in bits about a measurement of a_i from measurement of b_k is given by arguments of information theory [5]

$$I_{AB}(a_i,b_k) = \log_2\left(\frac{P_{AB}(a_i,b_k)}{P_A(a_i)P_B(b_k)}\right), \quad (3)$$

where the probability of observing a out of the set of all A is $P_A(a_i)$, and the probability of finding b in a measurement B is $P_B(b_i)$, and the joint probability of the measurement of a and b is $P_{AB}(a_i, b_k)$. The mutual information I of two measurements a_i and b_k is symmetric and non-negative, and equals to zero if only the systems are independent. The average mutual information between any value a_i from system A and b_k from B is the average over all possible measurements of $I_{AB}(a_i, b_k)$,

$$I_{AB}(\tau) = \sum_{a_i,b_k} P_{AB}(a_i,b_k) I_{AB}(a_i,b_k) \quad (4)$$

To place this definition to a context of observations from a certain physical system, let us think of the sets of measurements $s(n)$ as the A and of the measurements a time lag t later, $s(n$ + t), as B set. The average mutual information between observations at n and n + t is then

$$I_{AB}(\tau) = \sum_{a_i,b_k} P_{AB}(a_i,b_k) I_{AB}(a_i,b_k) \quad (5)$$

Now we have to decide what property of I(t) we should select, in order to establish which among the various values of t we should use in making the data vectors **y**(n). In ref. [13] it has been suggested, as a prescription, that it is necessary to choose that t where the first minimum of I(t) occurs. On the other hand, the autocorrelation coefficient failed to achieve zero, i.e. the autocorrelation function analysis not provides us with any value of t. Such an analysis can be certainly extended to values exceeding 1000, but it is known that an attractor cannot be adequately reconstructed for very large values of t. The mutual information function usually [5] exhibits an initial rapid decay (up to a lag time of about 10) followed more slow decrease before attaining near-saturation at the first minimum.

One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. If a time series under consideration have an n-dimensional Gaussian distribution, these statistics are theoretically equivalent as it is shown in Ref. [22]. The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process resulting in the vibrations amplitude level variations can be concluded.

2.3 Choosing embedding dimension. Correlation integral

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. In accordance with the embedding theorem, the embedding dimension, d_E, must be greater, or at least equal, than a dimension of attractor, d_A, i.e. $d_E > d_A$. In other words, we can choose a fortiori large dimension d_E, e.g. 10 or 15, since the previous analysis provides us prospects that the dynamics of our system is probably chaotic. However, two problems arise with working in dimensions larger than really required by the data and time-delay embedding [1,7,13,19]. First, many of computations for extracting interesting properties from the data require searches and other operations in R^d whose computational cost rises exponentially with d. Second, but more significant from the physical point of view, in the presence of noise or other high dimensional contamination of the observations, the extra dimensions are not populated by dynamics, already captured by a smaller dimension, but entirely by the contaminating signal. In too large an embedding space one is unnecessarily spending time working around aspects of a bad representation of the observations which are solely filled with noise. It is therefore necessary to determine the dimension d_A.

There are several standard approaches to reconstruct the attractor dimension (see, e.g., [1,7,23]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [23] is the most commonly used approach. According to this algorithm, the correlation integral is

$$C(r) = \lim_{N\to\infty} \frac{2}{N(n-1)} \sum_{\substack{i,j \\ (1\le i<j\le N)}} H\left(r - \|\mathbf{y}_i - \mathbf{y}_j\|\right) \qquad (6)$$

where H is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for u £ 0, r is the radius of sphere centered on $\mathbf{y}_i$ or $\mathbf{y}_j$, and N is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r\to 0 \\ N\to\infty}} \frac{\log C(r)}{\log r}, \qquad (7)$$

where d is correlation exponent that can be determined as the slop of line in the coordinates log $C(r)$ versus log r by a least-squares fit of a straight line over a certain range of r, called the scaling region.

If the correlation exponent attains saturation with an increase in the embedding dimension, then the system is generally considered to exhibit chaotic dynamics. The saturation value of the correlation exponent is defined as the correlation dimension (d_2) of the attractor. The nearest integer above the saturation value provides the minimum or optimum embedding dimension for reconstructing the phase-space or the number of variables necessary to model the dynamics of the system. On the other hand, if the correlation exponent increases without bound with increase in the embedding dimension, the system under investigation is generally considered stochastic. There are certain important limitations in the use of the correlation integral analysis in the search for chaos. For instance, the selection of inappropriate values for the parameters involved in the method may result in an underestimation (or overestimation) of the attractor dimension [24]. Consequently, finite and low correlation dimensions could be observed even for a stochastic process. To verify the results obtained by the correlation integral analysis, we use surrogate data method.

The method of surrogate data [1,7,19] is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data. This means that the surrogate data possess some of the properties, such as the mean, the standard deviation, the cumulative distribution function, the power spectrum, etc., but are otherwise postulated as random, generated according to a specific null hypothesis. Here, the null hypothesis consists of a candidate linear process, and the goal is to reject the hypothesis that the original data have come from a linear stochastic process. One reasonable statistics suggested by Theiler et al. [24] is obtained as follows. If we denote Q_{orig} as the statistic computed for the original time series and Q_{si} for ith surrogate series generated under the null hypothesis and let m_s and s_s denote, respectively, the mean and standard deviation of the distribution of Q_s, then the measure of significance S is given by

$$S = \frac{|Q_{orig} - \mu_s|}{\sigma_s}. \tag{8}$$

An S value of ~2 cannot be considered very significant, whereas an S value of ~10 is highly significant. To detect nonlinearity in the amplitude level data, the one hundred realizations of surrogate data sets were generated according to a null hypothesis in accordance to the probabilistic structure underlying the original data. Often, a significant difference in the estimates of the correlation exponents, between the original and surrogate data sets, can be observed. In the case of the original data, a saturation of the correlation exponent is observed after a certain embedding dimension value (i.e., 6), whereas the correlation exponents computed for the surrogate data sets continue increasing with the increasing embedding dimension. The high significance values of the statistic indicate that the null hypothesis (the data arise from a linear stochastic process) can be rejected and hence the original data might have come from a nonlinear process. It is worth consider another method for determining d_E that comes from asking the basic question addressed in the embedding theorem: when has one eliminated false crossing of the orbit with itself which arose by virtue of having projected the attractor into a too low dimensional space? By examining this question in dimension one, then dimension two, etc. until there are no incorrect or false neighbours remaining, one should be able to establish, from geometrical consideration alone, a value for the necessary embedding dimension. Such an approach was originally described by Kennel et al. [6]. Advanced version is presented in Ref. [16]

2.4 Lyapunov exponents

The LE are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local LE [7,25-29]. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of LE is considered a measure of the effect of perturbing the initial conditions of a dynamical system. Note that both positive and negative LE can coexist in a dissipative system, which is then chaotic. Since the LE are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of the LE, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, K, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive LE. The inverse of the Kolmogorov entropy is equal to an average predictability. Estimate of dimension of the attractor is provided by the Kaplan and Yorke conjecture:

$$d_L = j + \frac{\sum_{\alpha=1}^{j} \lambda_\alpha}{|\lambda_{j+1}|}, \tag{9}$$

where j is such that $\sum_{\alpha=1}^{j} \lambda_\alpha > 0$ and $\sum_{\alpha=1}^{j+1} \lambda_\alpha < 0$, and the LE l_a are taken in descending order. There are a few approaches to computing the LE. One of them computes the whole spectrum and is based on the Jacobi matrix of system [27]. In the case where only observations are given and the system function is unknown, the matrix has to be estimated from the data. In this case, all the suggested methods approximate the matrix by fitting a local map to a sufficient number of nearby points. In our work we use the method with the linear fitted map proposed by Sano and Sawada [27], although the maps with higher order polynomials can be also used. To calculate the spectrum of the

LE from the amplitude level data, one could determine the time delay t and embed the data in the four-dimensional space. In this point it is very important to determine the Kaplan-Yorke dimension and compare it with the correlation dimension, defined by the Grassberger-Procaccia algorithm. The estimations of the Kolmogorov entropy and average predictability can further show a limit, up to which the amplitude level data can be on average predicted.

3. Chaotic elements in dynamics of quantum and laser systems: Some illustrations

3.1. Non-linear analysis of chaotic oscillations in a grid of quantum generators

As the first illustration we present results of non-linear analysis of the chaotic oscillations in a grid o two autogenerators. Its regular and chaotic dynamics has been in details studied in many papers (see e.g. [2,14,31]. In Ref.[2] the time series for the characteristic vibration amplitude are presented in a case of two semiconductors lasers connected through general resonator. We use these data as input ones in the non-linear analysis of chaotic oscillations. Figure 1 presents the variations of the autocorrelation coefficient for the amplitude level. Autocorrelation function exhibits some kind of exponential decay up to a lag time of about 100 time units (sec).

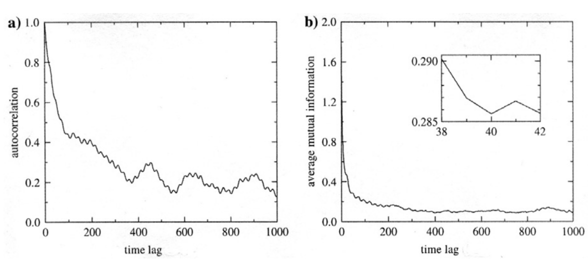

Figure 1. (a) Autocorrelation function and (b) average mutual information

Such an exponential decay can be an indication of the presence of chaotic dynamics in the process of the level variations. The autocorrelation coefficient failed to achieve 0, i.e. autocorrelation function analysis not provides with any value of t. Such an analysis can be certainly extended to values exceeding 1000, but it is known that an attractor cannot be adequately reconstructed for very large values of t. Figure 2 shows the relationship between the correlation exponent values and the embedding dimension values for original data set and mean values of the surrogate data sets as well as for one surrogate realization. Saturation value of the correlation exponent, i.e. correlation dimension of attractor, for the amplitude level series is about 3.5 and occurs at the embedding dimension value of 6. The low, non-integer correlation dimension value indicates the existence of low-dimensional chaos in the vibrations dynamics of the autogenerators. The dimension of the embedding phase-space is equal to the number of variables present in the evolution of the system dynamics. Our study indicate that to model the dynamics of process resulting in the amplitude level variations the minimum number of variables essential is equal to 4 and the number of variables sufficient is equal to 6.. To verify the results obtained by the correlation integral analysis, we use surrogate data method. It is method that makes using substitute data generated in accordance to probabilistic structure underlying the original data.

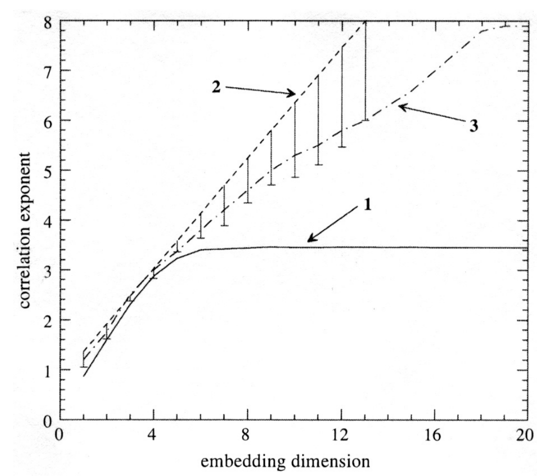

Figure 2. Relationship between correlation exponent and embedding dimension for vibrations amplitude level data for original time series (line 1), mean values of surrogate data sets (line 2), and one surrogate realization (line 3). Error bars indicate minimal values of correlation exponent among all realizations of surrogate data.

The surrogate data possess some of the properties, such as the mean, the standard deviation, the cumulative distribution function, the power spectrum, etc., but are otherwise postulated as random, generated according to a specific null hypothesis. The significance values (*S*) of correlation exponent are computed for each embedding dimension. The high significance values of the statistic indicate that the null hypothesis (the data arise from a linear stochastic process) can be rejected and hence the original data might have come from a nonlinear process. Figure 3 displays the percentage of false nearest neighbours that was determined for the amplitude level series, for phase-spaces reconstructed with embedding D from 1 to 20.

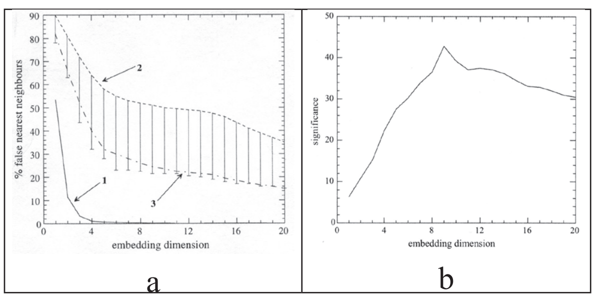

Figure 3. (a) Relationship between significance values of correlation dimension and embedding dimension; (b) Embedding dimension estimation by false nearest neighbour method for amplitude level data for original t series (line 1), mean values of surrogate data sets (2), one surrogate realization (3). Error bars show min % of false nearest neighbour among all realizations of surrogate data.

The percentage drops to almost zero at 4 or 5. This indicates that a 4- or 5-D phase-space is necessary to represent the dynamics (or unfold the attractor) of the amplitude level series. The mean percentage of false nearest neighbours computed for the surrogate data sets decreases steadily but at 20 is about 35%. Such a result seems to be in close agreement with that was obtained from the correlation integral analysis, providing further support to the observation made.

3.2. Non-linear analysis of chaotic self-oscillations in a laser system with absorbing cell

Here we consider a chaotic dynamics of a single-mode laser with the nonlinear absorption cell. This system can be used for the experimental observation of dynamic chaos. We consider a theoretical model of a single-mode laser resonator in which the reinforcement is placed along with a nonlinear absorbing medium. Each of the environments consists of identical two-level atoms. The gain and absorption lines are uniformly broadened and their centers align and coincide with one of the frequencies of the cavity. Such a model can describe the real system of five differential equations [31]:

$$
\begin{aligned}
d_e/d\tau &= -e + p_1 + p_2, \\
dp_1/d\tau &= -\delta_1(p_1 + em_1), \\
dp_2/d\tau &= -\delta_2(p_2 + em_2), \\
dm_1/d\tau &= -\rho_1(m_1 - m_{01} - ep_1), \\
dm_2/d\tau &= -\rho_2(m_2 - m_{02} - \beta ep_2).
\end{aligned}
\tag{10}
$$

Here, the index 1 refers to intensify, and the index of 2 - to an absorbing medium; *e*, p_1, p_2, m_1, m_2 are the dimensionless variables, *e* is an amplitude of the laser of the field, p_k is a polarization in the environment, m_k is the difference between the populations of the working levels; p_k and d_k are, respectively, the longitudinal and transverse relaxation rate, related to the half-width of the resonator $dw_p/2$, k=1,2; m_{0k} is the difference between the populations of the working levels in the absence of generation (m_{01}<0, m_{02}>0), *b* -the ratio of the coefficients of saturation of the absorbing and amplifying media; $t=tdw_p/2$ is the dimensionless time. Attractor of the system can be as invariant with respect to this change (let's call this attractor "symmetrical") and non-invariant ("asymmetric"). In the latter case certainly, there are two attractor into each other after this change. In fig. 4 we present the results of the numerical simulation for the system (10) [31].

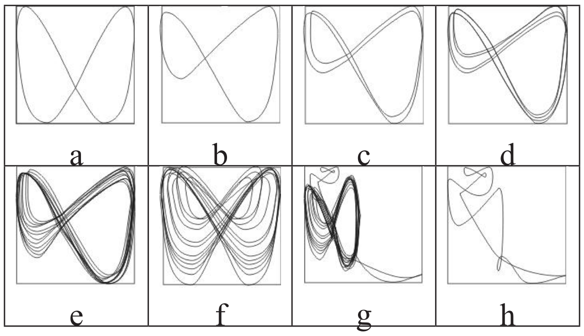

Figure 4. Projections of the phase trajectories for different values of the parameter h: a - 1.7000, b - 1.8200, c - 1.8350, d - 1.8385, e - 1.8500, f - 1.8800, g - 1.9000, h – 1.9500

Strange attractors occur as a result of the sequence of bifurcations of solutions of (10), the first of which is the Hopf bifurcation of stationary solutions with zero intensity of the laser field. This bifurcation occurs when $\eta = \delta_2[1 + (\delta_2)(1 + \delta_1 + m_{02})/\delta_1(1 + \delta_1)]$, if $h<m_{02}$. Our analysis shows that the Hopf bifurcation occurs at moderate values *h*, if the relative width of the absorption line d_2 is quite small, and the relative width of the gain line d_1 is quite large. The numerical calculation shows that in order to get the chaotic lasing it is necessary the following: to saturate the absorber should be saturated stronger than the amplifier (*b*>1). At low *b* the limit cycles generated from the stationary solutions with the zeroth intensity is stable up to very large values of *h*. In table 1 we present the computed values of the LE l_1-l_6 in the descending order and the Kolmogorov entropy (K). Another important illustration is non-linear analysis of the chaotic self-ocsillations in the backward wave tube (device for generating electromagnetic vibrations of the HF range.)

In Refs.[2] there have been presented the temporal dependences of the output signal amplitude, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and for developed chaos at large values of the dimensionless length parameter.

Table 1

Numerical parameters of the chaotic regimes in a 1-mode laser with absorbing cell: l_1-l_6 are the Lyapunov exponents in descending order, K – Kolmogorov entropy (our data)

Regime	λ_1	λ_2	λ_3
Weak chaos	0.175	-0.0001	–0.0003
Strong chaos	0.542	0.203	-0.0001
Regime	λ_4	λ_5	λ_6
Weak chaos	–0.244	–	–
Strong chaos	–0.0004	–0.067	–0.188

4. Conclusions

Here we present the results of computing nonlinear chaotic dynamics of some quantum and laser systems with using advanced techniques [8-17,30]. The correlation dimension method provided a low fractal-dimensional attractor thus suggesting a possibility of the existence of chaotic behaviour. The method of surrogate data, for detecting nonlinearity, provided significant differences in the correlation exponents between the original data series and the surrogate data sets. This finding indicates that the null hypothesis (linear stochastic process) can be rejected. It has been shown that the systems exhibit a nonlinear behaviour with elements of a low-dimensional chaos. The LE analysis does support this conclusion.

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References

1. H.Abarbanel, R.Brown, J.Sidorowich and L.Tsimring, Rev.Mod.Phys. (1993) 5, 1331.
2. A.Vedenov, A.Ezhov and E. Levchenko,

in Non-linear waves. Structure and bifurcations, ed. by A. Gaponov-Grekhov and M. Rabinovich, (Nauka, Moscow, 1997), pp.53-69.
3. M. Gutzwiller, Chaos in Classical and Quantum Mechanics (N.-Y., Springer, 1999), 720 p.
4. E. Ott, Chaos in dynamical systems, (Cambridge: Cambridge Univ.Press, 2002), 490p.
5. R. Gallager, Information theory and reliable communication (N.-Y.,Wiley, 1986).
6. D. Ullmo, Rep. Prog. Phys. 71, 026001 (2008).
7. A.V. Glushkov, Modern theory of a chaos, (Odessa, Astroprint, 2008), 450p.
8. A. Glushkov, O. Khetselius, S. Brusentseva, P. Zaichko and V. Ternovsky, in Adv. in Neural Networks, Fuzzy Systems and Artificial Intelligence, Series: Recent Adv. in Computer Engineering, vol. 21, ed. by J.Balicki (WSEAS, Gdansk, 2014), pp.69-75.
9. A. Glushkov, A. Svinarenko, V. Buyadzhi, P. Zaichko and V. Ternovsky, in: Adv.in Neural Networks, Fuzzy Systems and Artificial Intelligence, Series: Recent Adv. in Computer Engineering, vol.21, ed. by J.Balicki (WSEAS, Gdansk, 2014), pp.143-150.
10. A.V. Glushkov, O.Yu. Khetselius, A.A. Svinarenko, G.P. Prepelitsa, Energy approach to atoms in a Laser Field and Quantum Dynamics with Laser Pulses of Different Shape, In: Coherence and Ultrashort Pulsed Emission, ed. by F.J. Duarte (Intech, Vienna, 2011), 101-130.
11. A.V. Glushkov, V.N. Khokhlov, I.A. Tsenenko, Atmospheric teleconnection patterns: wavelet analysis, Nonlin. Proc.in Geophys. 11, 285 (2004).
12. A.V. Glushkov, V.N. Khokhlov, N.S. Loboda, N.G. Serbov, K. Zhurbenko, Stoch. Environ. Res. Risk Assess. (Springer) 22, 777 (2008).
13. A.V. Glushkov, V.N. Khokhlov, N.S. Loboda, Yu.Ya. Bunyakova, Atm. Environment (Elsevier) 42, 7284 (2008).
14. A. Glushkov, Y. Bunyakova, A. Fedchuk, N. Serbov, A. Svinarenko and I. Tsenenko, Sensor Electr. and Microsyst. Techn. 3(1), 14 (2007).
15. A.V. Glushkov, V. Kuzakon, V. Ternovsky and V. Buyadzhi, in: Dynamical Systems Theory, vol.T1, ed by J. Awrejcewicz, M. Kazmierczak, P. Olejnik, and J. Mrozowski (Lodz, Polland, 2013), P.461-466.
16. A.V.Glushkov, G. Prepelitsa, A. Svinarenko and P. Zaichko, in: Dynamical Systems Theory, vol.T1, ed by J. Awrejcewicz, M. Kazmierczak, P. Olejnik, and J. Mrozowski (Lodz, Polland, 2013), pp.P.467-487.
17. E. N. Lorenz, Journ. Atm. Sci. 20, 130 (1993).
18. N. Packard, J. Crutchfield, J. Farmer and R. Shaw, Phys.Rev.Lett. 45, 712 (1998).
19. M. Kennel, R. Brown and H. Abarbanel, Phys.Rev.A. 45, 3403 (1992).
20. F. Takens in: Dynamical systems and turbulence, ed by D. Rand and L. Young (Springer, Berlin, 1991), pp.366–381
21. R. Mañé, in: Dynamical systems and turbulence, ed by D. Rand and L. Young (Springer, Berlin, 1999), pp.230–242.
22. M. Paluš, E.Pelikán, K. Eben, P. Krejčíř and P. Juruš, in: Artificial Neural Nets and Genetic Algorithms, ed. V. Kurkova (Springer, Wien, 2001), pp. 473-476.
23. P. Grassberger and I. Procaccia, Physica D. 9, 189 (1993).
24. J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, J. Farmer, Phys.D. 58, 77 (1992).
25. A. Fraser and H. Swinney, Phys Rev A. 33, 1134 (1996).
26. J. Havstad and C. Ehlers, Phys.Rev.A. 39, 845 (1999).
27. M. Sano and Y. Sawada, Phys Rev.Lett., 55, 1082 (1995).
28. T. Schreiber, Phys.Rep. 308, 1 (1999).

29. H. Schuste, Deterministic Chaos: An Introduction (Wiley, N.-Y., 2005), 312 p.
30. A. Glushkov, G. Prepelitsa, S. Dan'kov, V. Polischuk and A. Efimov, Journ. Tech. Phys. 38, 219 (1997).
31. A.G. Vladimirov and E.E. Fradkin, Optics and Spectr. 67, 219 (1999).
32. S.P. Kuznetsov and D.I. Trubetskov, Izv. Vuzov. Ser. Radiophys. 48, 1 (2004).

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NONLINEAR DYNAMICS OF QUANTUM AND LASER SYSTEMS WITH ELEMENTS OF A CHAOS

Abstract Nonlinear chaotic dynamics of the quantum and laser systems is studied with using advanced techniques such as a wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, the Lyapunov exponent's analysis, and surrogate data method. The detailed analysis of the oscillations in a grid of two autogenerators and single-mode laser with the nonlinear absorption cell shows that the systems exhibit a nonlinear behaviour with elements of a low-dimensional chaos.

Key words: Quantum and laser systems -chaos -non-linear analysis

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НЕЛИНЕЙНАЯ ДИНАМИКА КВАНТОВЫХ И ЛАЗЕРНЫХ СИСТЕМ С ЭЛЕМЕНТАМИ ХАОСА

Резюме

Изучается нелинейные хаотическая динамика квантовых и лазерных систем с использованием техники нелинейного анализа, включающей вейвлет-анализ, мульти-фрактальный формализм, метод взаимной информации, метод корреляционного интеграла, алгоритм ложных ближайших соседей, метод показателей Ляпунова, метод суррогатных данных и др. Детальный анализ осцилляций в системе двух автогенераторов и одномодового лазера с нелинейной поглощающей ячейкой показывает, что в динамике указанных систем имеет место нелинейное поведение с элементами низко-размерного хаоса.

Ключевые слова: Квантовые и лазерные систем, хаос, нелинейный анализ

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НЕЛІНІЙНА ДИНАМІКА КВАНТОВИХ І ЛАЗЕРНИХ СИСТЕМ З ЕЛЕМЕНТАМИ ХАОСА

Резюме

Вивчається нелінійні хаотична динаміка квантових і лазерних систем з використанням техніки нелінійного аналізу, що включає вейвлет-аналіз, мульті-фрактальний формалізм, метод взаємної інформації, метод кореляційного інтеграла, алгоритм помилкових найближчих сусідів, метод показників Ляпунова, метод сурогатних даних і ін. Докладний аналіз осциляцій в системі двох автогенераторів і одномодового лазеру з нелінійною поглинаючою коміркою показує, що в динамиці вказаних систем має місце нелінійна поведінка з елементами низько-розмірного хаосу.

Ключові слова: Квантові та лазерні систем, хаос, нелінійний аналіз