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# CALCULATING THE RADIATIVE VACUUM POLARIZATION CONTRIBUTION TO THE ENERGY SHIFT OF SOME TRANSITIONS IN PIONIC AND KAONIC NITROGEN

Calculating the radiative contribution due to the vacuum polarization effect to energy value for some transitions in pionic and kaonic nitrogen has been carried out using the modified Uehling-Serber potential. The master equation for computing spectrum of the kaonic atoms is the Klein-Gordon-Fock equation. The vacuum polarization contribution to the 8-7 transition energy in kaonic nitrogen is calculated

This paper goes on our work on estimating the vacuum-polarization contributions to the transition energies of the hadronic atoms that is of a great importance especially in the last years (look below) [1-15]. Here we study pionic and kaonic nitrogen. Let us remind [13] that due to the significant progress in the modern experimental technologies now a great interest attracts studying spectra of heavy and super heavy elements atoms, exotic atomic systems, including hadronic and leptonic atoms [1-15]. Especial problem is connected with précised calculating the radiative corrections to the transition energies of the low-Z exotic (pionic, kaonic, muonic) atoms, namely, hydrogen and deuterium. Naturally, it is provided by necessity of further developing the modern as atomic and as nuclear theories. From the other side, detailed information about spectra of the exotic atomic systems (kaonic, pionic, muonic atoms) can be very useful under construction of the new X-ray standards. One could remind a great importance of the muonic chemistry, muonic spectroscopy. Very attractive perspective of the thermonuclear fission through the mechanism of the muonic catalysis is still interesting and widely studied.

The standard Dirac approach is traditionally used as starting basis in calculations of the heavy ions [2]. The problem of accounting the radiative corrections, in particular, self-energy part of the Lamb shift and vacuum polarization contribution is mostly treated with using the expansions on the natural physical parameters $1/Z$, $\alpha Z$ ($\alpha$ is fine structure constant) [5,10]. It permits evaluations of the relative contributions of different expansion energy terms: non-relativistic, relativistic ones, as functions of Z. For high Z (Z is a nuclear charge) it should be necessary to account for the high-order QED corrections and the nuclear finite size correction etc [1-3,10-12,16]. Further improvement of this method in a case of the heavy ions is linked with using gauge invariant procedures of generating relativistic orbital basises and more correct treating nuclear and QED effects [1-3]. In a case of the low-Z exotic atomic systems such as an exotic hydrogen (deuterium) a great interest attracts estimation of the radiative, in particular, vacuum polarization, correction. In refs. [17-19] it has been proposed a precise scheme to calculating spectra of heavy systems with account of nuclear and radiative effects, based on the relativistic many-body perturbation theory (see also

[3]) and advanced effective procedures for accounting the radiative corrections.

In this paper we present the results of calculating the contribution due to the vacuum polarization effect to energy shift for some transitions in pionic and kaonic nitrogen. The obtained result is compared with calculation data by Indelicato [19]. The details of the calculation procedure have been presented earlier, and here we consider only the key topics. The calculation of the radiative vacuum polarization shift in the pionic deuterium should be performed using the Dirac approximation as a zeroth one. Further, the expectation value of the radiative vacuum polarization operator gives the corresponding correction. The total electromagnetic interaction potential:

$$V(r)=V_n(r)+U(r). \qquad (1)$$

includes the electrical $V_n$ and polarization $U$(r) potentials of a nucleus with accounting the finite size correction. As usually, the Coulomb potential for spherically symmetric density $\rho(r|R)$ can be written as follows:

$$V_n(r|R)=-\left((1/r)\int_0^r dr' r'^2 \rho(r'|R)+\int_r^\infty dr' r' \rho(r'|R)\right). \qquad (2)$$

The details of the determination of this potential can be , for example, found in ref. [21,22]. The vacuum polarization part is usually accounted in the first PT order by using the Uehling potential [1,8,16,17]:

$$U(r)=-\frac{2\alpha}{3\pi r}\int_1^\infty dt \exp(-2rt/\alpha Z)(1+1/2t^2)\frac{\sqrt{t^2-1}}{t^2}\equiv$$

$$=-\frac{2\alpha}{3\pi r}C(g) \qquad (3)$$

$$g=\frac{r}{\alpha Z}. \qquad (4)$$

The corresponding expectation value of this operator gives the corresponding vacuum polarization correction. In the scheme [12] this potential is approximated by quite precise analytical function (see details in refs. [3,16,17]). The most advanced version of the such potential ($C\rightarrow\tilde{\tilde{C}}$) is presented as follows:

$$\tilde{\tilde{C}}(g)=\tilde{C}_1(g)\tilde{\tilde{C}}_2(g)/\left(\tilde{C}_1(g)+\tilde{\tilde{C}}_2(g)\right),$$

$$\tilde{\tilde{C}}_2(g)=\tilde{C}_2(g)\,f(g),$$

$$\tilde{C}_2(g)=-1.8801\exp(-g)/g^{3/2}$$

$$\tilde{C}_1(g)=\mathrm{h}(g/2)+1.410545-1.037837g,$$

$$f(g)=\left((1.1024/g-1.3361)/g+0.8027\right)$$

The using this formula permits one to decrease the calculation errors for this term down to ~0.1%. Error of usual calculation scheme is~10%.

We carried out the calculation of the vacuum polarization contribution to the energy shift for 8k-7 transition in kaonic nitrogen It should be noted that the energy levels of exotic (kaonic) muonic atoms are very sensitive to effects of QED, nuclear structure and recoil since the kaon is heavier than the electron. As usually the fundamental constants from the CODATA 1998 are used in the numerical calculations.

We have evaluated the modified Uehling-Serber potential expectation values and obtained the value for the vacuum-polarization correction: 1.1783 eV. It is interesting to compare this result with the result by Indelicato, namely, the total vacuum polarization contribution: 1.1789 eV and from ref. [23]: 1,1778 eV. Further, let us present the data for the main Coulomb contribution from [8,24] - 2968.4565 eV and from ref. [23] - 2968.4492eV. Our estimate: 2968.4481eV. Further we have evaluated the modified Uehling-Serber potential expectation values and obtained the value for the vacuum-polarization correction into the 5f-4d transition energy in the pionic nitrogen. Our value for the vacuum-polarization correction: 2,9467eV. It is interesting to compare this result with the result by Serga [25] 3,216 eV. It should be noted that the last result is received with account of the Wichman-Kroll and Kallen-Sabry corrections. In any case, the physically reasonable agreement can be easily explained by the fact that pionic and kaonic nitrogen is the low-Z atomic system and in this case the expansion on the parameter $\alpha Z$ works sufficiently well.

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**Abstract**

Calculating the radiative contribution due to the vacuum polarization effect to energy value for some transitions in pionic and kaonic atoms has been carried out using the modified Uehling-Serber potential. The master equation for computing spectrum of the kaonic atoms is the Klein-Gordon-Fock equation. The vacuum polarization contribution to the 8-7 transition energy in kaonic nitrogen is calculated.

**Key words**: kaonic nitrogen, radiative corrections

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**РАСЧЕТ РАДИАЦИОННОГО ВКЛАДА ЗА СЧЕТ ЭФФЕКТА ПОЛЯРИЗАЦИИ ВАКУУМА В СДВИГ ЭНЕРГИИ РЯДА ПЕРЕХОДАОВ В ПИОННОМ И КАОННОМ АЗОТЕ**

**Резюме**

Проведен расчет радиационного вклада за счет эффекта поляризации вакуума в величину энергии ряда переходов в пионом и каонном атомах с использованием модифицированного потенциала Юлинга-Сербера. Базисным для расчета спектра каонных атомов является уравнение Клейна-Гордона-Фока. Рассчитан вклад за счет поляризации вакуума в энергию 8-7 перехода в каонном азоте.

**Ключевые слова**: каонный азот, радиационные поправки

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# РОЗРАХУНОК РАДІАЦІЙНОГО ВНЕСКУ ЗА РАХУНОК ЕФЕКТУ ПОЛЯРИЗАЦІЇ ВАКУУМУ У ЗСУВ ЕНЕРГІЇ ДЕКОТРИХ ПЕРЕХОДІВ У ПІОННОМУ І КАОННОМУ АЗОТЕ

**Резюме**

Виконано розрахунок радіаційного внеску за рахунок ефекту поляризації вакууму у величину енергії декотрих переходів у піонному і каонному атомах з використанням модифікованого потенціалу Юлінга-Сербера . Базисным для розрахунку спектра каонних атомів є рівняння Клейна-Гордона-Фока. Розраховано внесок за рахунок поляризації вакууму в энергію 8-7 переходу у каонному азоті.

**Ключові слова**: каонний азот, радіаційні поправки