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## **SIMULATION CHAOTIC DYNAMICS OF COMPLEX SYSTEMS AND DEVICES WITH USING CHAOS THEORY, GEOMETRIC ATTRACTORS, AND QUANTUM NEURAL NETWORKS**

Nonlinear simulation and forecasting chaotic evolutionary dynamics of complex systems can be effectively performed using the concept of compact geometric attractors. We are developing a new approach to analyze and forecasting complex systems evolutionary dynamics based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation.

In recent years a considerable number of works, including an analysis from the perspective of the theory of dynamical systems and chaos, fractal sets, is devoted to time series analysis of dynamical characteristics of physics and other systems [1-11]. In a series of papers [10-20] the authors have attempted to apply some of these methods in a variety of the physical, geophysical, hydrodynamic problems. In connection with this, there is an extremely important task on development of new, more effective approaches to the nonlinear modelling and prediction of chaotic processes in different complex systems.

In this work nonlinear simulation and forecasting chaotic evolutionary dynamics of complex systems are carried out using the concept of compact geometric attractors [17-20]. We are developing a new approach to analyze complex system dynamics based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation

The basic idea of the construction of our approach to prediction of chaotic processes in complex systems is in the use of the traditional concept of a compact geometric attractor in which evolves the measurement data, plus the implementation of neural network algorithms. The existing so far in

the theory of chaos prediction models are based on the concept of an attractor, and are described in a number of papers (e.g. [1-8]).

From a mathematical point of view, it is a fact that in the phase space of the system an orbit continuously rolled on itself due to the action of dissipative forces and the nonlinear part of the dynamics, so it is possible to stay in the neighborhood of any point of the orbit  $y(n)$  other points of the orbit  $y^r(n)$ ,  $r = 1, 2, \dots, N_B$ , which come in the neighborhood  $y(n)$  in a completely different times than  $n$ . Of course, then one could try to build different types of interpolation functions that take into account all the neighborhoods of the phase space and at the same time explain how the neighborhood evolve from  $y(n)$  to a whole family of points about  $y(n+1)$ . Use of the information about the phase space in the simulation of the evolution of some physical (geophysical etc.) process in time can be regarded as a fundamental element in the simulation of random processes.

In terms of the modern theory of neural systems, and neuro-informatics (e.g. [11]), the process of modelling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations (miemo-dynamic equations). Imitating the further evolution of a com-

plex system as the evolution of a neural network with the corresponding elements of the self-study, self-adaptation, etc., it becomes possible to significantly improve the prediction of evolutionary dynamics of a chaotic system. Considering the neural network with a certain number of neurons, as usual, we can introduce the operators  $S_{ij}$  synaptic neuron to neuron  $u_i, u_j$ , while the corresponding synaptic matrix is reduced to a numerical matrix strength of synaptic connections:  $W = || w_{ij} ||$ . The operator is described by the standard activation neuro-equation determining the evolution of a neural network in time:

$$s'_i = \text{sign}\left(\sum_{j=1}^N w_{ij} s_j - \theta_i\right) \quad (1)$$

where  $1 < i < N$ .

From the point of view of the theory of chaotic dynamical systems, the state of the neuron (the chaos-geometric interpretation of the forces of synaptic interactions, etc.) can be represented by currents in the phase space of the system and its topological structure is obviously determined by the number and position of attractors. To determine the asymptotic behavior of the system it becomes crucial a information aspect of the problem, namely, the fact of being the initial state to the basin of attraction of a particular attractor.

Modelling each physical attractor by a record in memory, the process of the evolution of neural network, transition from the initial state to the (following) the final state is a model for the reconstruction of the full record of distorted information, or an associative model of pattern recognition is implemented. The domain of attraction of attractors are separated by separatrices or certain surfaces in the phase space. Their structure, of course, is quite complex, but mimics the chaotic properties of the studied object. Then, as usual, the next step is a natural construction parameterized nonlinear function  $F(x, a)$ , which transforms:

$$y(n) \rightarrow y(n+1) = F(y(n), a),$$

and then to use the different (including neural network) criteria for determining the parameters  $a$  (see below). The easiest way to implement this program is in considering the original local neighborhood, enter the model(s) of the process occur-

ring in the neighborhood, at the neighborhood and by combining together these local models, designing on a global nonlinear model. The latter describes most of the structure of the attractor.

Although, according to a classical theorem by Kolmogorov-Arnold-Moser, the dynamics evolves in a multidimensional space, the size and the structure of which is predetermined by the initial conditions, this, however, does not indicate a functional choice of model elements in full compliance with the source of random data. One of the most common forms of the local model is the model of the Schreiber type [3] (see also [17-20]).

Nonlinear modelling of chaotic processes can be based on the concept of a compact geometric attractor, which evolve with measurements. Since the orbit is continually folded back on itself by the dissipative forces and the non-linear part of the dynamics, some orbit points  $y^r(k), r = 1, 2, \dots, N_B$  can be found in the neighbourhood of any orbit point  $y(k)$ , at that the points  $y^r(k)$  arrive in the neighbourhood of  $y(k)$  at quite different times than  $k$ . Then one could build the different types of interpolation functions that take into account all the neighborhoods of the phase space, and explain how these neighborhoods evolve from  $y(n)$  to a whole family of points about  $y(n+1)$ . Use of the information about the phase space in modelling the evolution of the physical process in time can be regarded as a major innovation in the modelling of chaotic processes.

This concept can be achieved by constructing a parameterized nonlinear function  $F(x, a)$ , which transform  $y(n)$  to  $y(n+1)=F(y(n), a)$ , and then using different criteria for determining the parameters  $a$ . Further, since there is the notion of local neighborhoods, one could create a model of the process occurring in the neighborhood, at the neighborhood and by combining together these local models to construct a global nonlinear model that describes most of the structure of the attractor.

As shown Schreiber [3], the most common form of the local model is very simple:

$$s(n + \Delta n) = a_0^{(n)} + \sum_{j=1}^{d_A} a_j^{(n)} s(n - (j-1)\tau) \quad (2)$$

where  $\Delta n$  - the time period for which a forecast .

The coefficients  $a_j^{(k)}$ , may be determined by a least-squares procedure, involving only

points  $s(k)$  within a small neighbourhood around the reference point. Thus, the coefficients will vary throughout phase space. The fit procedure amounts to solving  $(d_A + 1)$  linear equations for the  $(d_A + 1)$  unknowns. When fitting the parameters  $a$ , several problems are encountered that seem purely technical in the first place but are related to the nonlinear properties of the system. If the system is low-dimensional, the data that can be used for fitting will locally not span all the available dimensions but only a subspace, typically. Therefore, the linear system of equations to be solved for the fit will be ill conditioned. However, in the presence of noise the equations are not formally ill-conditioned but still the part of the solution that relates the noise directions to the future point is meaningless. Other modelling techniques are described, for example, in [3,10, 17-20].

Assume the functional form of the display is selected, wherein the polynomials used or other basic functions. Now, we define a characteristic which is a measure of the quality of the curve fit to the data and determines how accurately match  $y(k+1)$  with  $F(y(k), a)$ , calling it by a local deterministic error:

$$\varepsilon_D(k) = y(k+1) - F(y(k), \mathbf{a}). \quad (3)$$

The cost function for this error is called  $W(\varepsilon)$ . If the mapping  $F(y, a)$ , constructed by us, is local, then one has for each adjacent to  $y(k)$  point,  $y^{(r)}(k)$  ( $r = 1, 2, \dots, N_B$ ),

$$\varepsilon_D^{(r)}(k) = y(r, k+1) - F(y^{(r)}(k), \mathbf{a}), \quad (4)$$

where  $y(r, k+1)$  - a point in the phase space which evolves  $y(r, k)$ . To measure the quality of the curve fit to the data, the local cost function is given by

$$W(\varepsilon, k) = \frac{\sum_{r=1}^{N_B} |\varepsilon_D^{(r)}(k)|^2}{\sum_{r=1}^{N_B} [y(k) - \langle y(r, k) \rangle]^2} \quad (5)$$

and the parameters identified by minimizing  $W(\varepsilon, k)$ , will depend on  $a$ .

Furthermore, formally the neural network algorithm is launched, in particular, in order to make training the neural network system equivalent to the reconstruction and interim forecast the state

of the neural network (respectively, adjusting the values of the coefficients). The starting point is a formal knowledge of the time series of the main dynamic parameters of a chaotic system, and then to identify the state vector of the matrix of the synaptic interactions  $\|w_{ij}\|$  etc. Of course, the main difficulty here lies in the implementation of the process of learning neural network to simulate the complete process of change in the topological structure of the phase space of the system and use the output results of the neural network to adjust the coefficients of the function display.

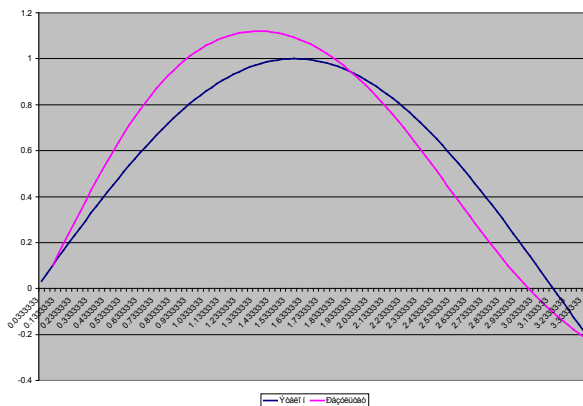
Further we consider implemetatiom of the quantum neural networks algorithm into general scheme of studying chaotic dynamics. The basic aspects of theory of the photon echo based neural networks are stated previously (see, for example, [10,11, 18,21,22]). So here we mention only the essential elements. Photon echo is a nonlinear optical effect, in fact this is the phenomenon of the four wave interaction in a nonlinear medium with a time delay between the laser pulses. One promising approach to the realization of an quantum neural network is proposed in refs. [11,18]. We have used a software package for numerical modeling of the dynamics of the photon echo neural network, which imitates evolutionary dynamics of the complex system. It has the following key features: multi-layering, possibility of introducing training, feedback and controlled noise. There are possible the different variants of the connections matrix determination and binary or continuous sigmoid response (and so on) of the model neurons. In order to imitate a tuition process we have carried out numerical simulation of the neural networks for recognizing a series of patterns (number of layers  $N=5$ , number of images  $p=640$ ; the error function:

$$SSE = \sum_{p=1}^{p_{\max}} \left\{ \sum_{k=1}^{k_{\max}} [t(p, k) - O(p, k)]^2 \right\}, \quad (6)$$

where  $O(p, k)$  - neural networks output  $k$  for image  $p$  and  $t(p, k)$  is the trained image  $p$  for output  $k$ ;  $SSE$  is determined from a procedure of minimization; the output error is  $RMS = \sqrt{SSE/P_{\max}}$ ; As neuronal function there is used function of the form:  $f(x) = 1/[1 + \exp(-\delta x)]$ . In our calculation there is tested the function  $f(x, T) = \exp[(xT)^4]$  too.

The result of the PC simulation (with using our neural networks package NNW-13-2003 [11]) of

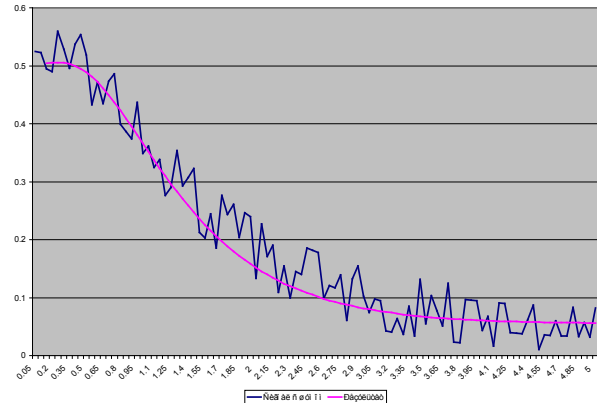
dynamics of the quantum multilayer neural networks with the input sinusoidal pulses is listed in fig.1. Fig. 2 demonstrates the results of modeling the dynamics of multilayer neural network for the case of noisy input sequence. The input signal was the Gaussian-like pulse with adding a noise with intensity D. At a certain value of the parameter D (the variation interval .0001-0.0040 ) the network training process and signal playback is optimal. The optimal value of D is 0.0017 . A coherency of input and output is optimal for the indicated optimal noise level. Thus, a stochastic resonance effect is in fact discovered in our PC experiment. In our view, this phenomenon is apparently typical for the neural network system. Obviously, one should search for the same effect for human tuition process. Analysis of the PC experiment results allows to make conclusion about sufficiently high-quality processing the input signals of very different shapes and complexity by a photon echo based neural network.



**Fig. 1. The results of modeling the dynamics of multilayer neural networks with sinusoidal input pulse.**

The most fundamental feature of the approach in development is combined using elements of of a chaos theory, concept of a compact geometric attractor, and one of the neural network algorithms, or, in a more general definition of a model of artificial intelligence. The meaning of the latter is precisely the application of neural network to simulate the evolution of the attractor in phase space, and training most neural network to predict (or rather, correct) the necessary coefficients of the parametric form of functional display. Using phase space information on the evolution in

time and results of the of quantum neural network modelling techniques can be considered as one of the fundamentally approaches in forecasting chaotic dynamics of the really very complex systems.



**Fig. 2. The results of modeling the dynamics of multilayer neural for the case of noisy input sequence.**

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This article has been received within 2014



UDC 539.182

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### **Abstract**

Nonlinear simulation and forecasting chaotic evolutionary dynamics of complex systems can be effectively performed using the concept of compact geometric attractors. We are developing a new approach to analyze and forecasting complex systems evolutionary dynamics based on the concept of geometric attractors, chaos theory methods and algorithms for quantum neural network simulation

**Key words:** geometric attractor conception, quantum neural networks, chaotic dynamics

УДК 539.182

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## **МОДЕЛИРОВАНИЕ ХАОТИЧНОЙ ДИНАМИКИ СЛОЖНЫХ СИСТЕМ И ПРИБОРОВ С ИСПОЛЬЗОВАНИЕМ ТЕОРИИ ХАОСА, ГЕОМЕТРИЧЕСКИХ АТТРАКТОРОВ И КВАНТОВЫХ НЕЙРОСЕТЕЙ**

### **Резюме**

Нелинейное моделирование и прогнозирование хаотических эволюционных динамик сложных систем может быть эффективно выполнено с использованием концепции компактных геометрических аттракторов. Мы развиваем эффективный подход для анализа и прогнозирования нелинейной эволюционной динамики сложных систем, основанный на концепции геометрических аттракторов, методов теории хаоса и алгоритмов для моделирования квантовой нейронной сети.

**Ключевые слова:** концепция геометрического аттрактора, квантовые нейронные сети, хаотическая динамика

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## **МОДЕЛЮВАННЯ ХАОТИЧНОЇ ДИНАМІКИ СКЛАДНИХ СИСТЕМ І ПРИЛАДІВ З ВИКОРИСТАННЯМ ТЕОРІЇ ХАОСУ, ГЕОМЕТРИЧНИХ АТТРАКТОРІВ І КВАНТОВИХ НЕЙРОМЕРЕЖ**

### **Резюме**

Нелінійне моделювання і прогнозування хаотичних еволюційних динамік складних систем може бути ефективно виконане з використанням концепції компактних геометричних аттракторів. Ми розвиваємо ефективний підхід для аналізу й прогнозування нелінійної еволюційної динаміки складних систем, оснований на концепції геометричних аттракторів, методів теорії хаосу і алгоритмів для моделювання квантової нейронної мережі.

**Ключевые слова:** концепція геометричного аттрактора, квантові нейронні мережі, хаотична динаміка