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RELATIVISTIC THEORY OF SPECTRA OF HEAVY PIONIC ATOMS WITH ACCOUNT OF STRONG PION-NUCLEAT INTERACTION EFFECTS: NEW DATA FOR ^{175}Lu, ^{205}Tl, ^{202}Pb,

New relativistic method of the Klein-Gordon-Fock equation with an generalized pion-nuclear potential is used to determine the transition energies with accounting for the strong pion-nuclear interactions effects in spectroscopy of some heavy pionic atoms. As example, there is carried out studying the Coulomb, nuclear and strong interaction contributions into the 4f-3d, 5g-4f transitions energies for the ^{175}Lu, ^{205}Tl, ^{202}Pb pionic atoms.

1. Introduction

In papers [1-3] we have developed a new relativistic method of the Klein-Gordon-Fock equation with an generalized pion-nuclear potential to determine transition energies in spectroscopy of light, middle and heavy pionic atoms with accounting for the strong interaction effects. In this paper, which goes on our studying on spectroscopy of pionic atoms, we firstly applied method [1-3] to calculating transition energies in a set of the heavy pionic atoms, in particular, atoms of ^{175}Lu, ^{205}Tl, ^{202}Pb, with accounting for the strong pion-nuclear interaction effects.

Following [1-3], let us remind that spectroscopy of hadron atoms has been used as a tool for the study of particles and fundamental properties for a long time. Exotic atoms are also interesting objects as they enable to probe aspects of atomic and nuclear structure that are quantitatively different from what can be studied in electronic or "normal" atoms. At present time one of the most sensitive tests for the chiral symmetry breaking scenario in the modern hadron's physics is provided by studying the exotic hadron-atomic systems. Nowadays the transition energies in pionic (kaonic, muonic etc.) atoms are measured with an unprecedented precision and from studying spectra of the hadronic atoms it is possible to investigate the strong interaction at low energies measuring the energy and natural width of the ground level with a precision of few meV [1-10].

The strong interaction is the reason for a shift in the energies of the low-lying levels from the purely electromagnetic values and the finite lifetime of the state corresponds to an increase in the observed level width. The possible energy shifts caused by the pion-induced fluorescence X-rays were checked in the measurement of the pion beams at PSI in Switzerland. For a long time the similar experimental investigations have been carried out in the laboratories of Berkley, Virginia (USA), CERN (Switzerland).

The most known theoretical models to treating the hadronic (pionic, kaonic, muonic, antiprotonic etc.) atomic systems are presented in refs. [1-5,7,8]. The most difficult aspects of the theoretical modeling are reduced to the correct description of pion-nuclear strong interaction [1-3] as the electromagnetic part of the problem is reasonably accounted for. Besides, quite new aspect is linked with the possible, obviously, very tiny electroweak and hyperfine interactions.

2. Relativistic approach to pionic atoms spectra

As the basis's of a new method has been published, here we present only the key topics of an approach [1-3]. All available theoretical models to treating the hadronic (kaonic, pionic) atoms are naturally based on the using the Klein-Gordon-Fock equation [2,5], which can be written as follows :

(1)

where c is a speed of the light, *h* is the Planck constant, and $\Psi_0(x)$ is the scalar wave function of the space-temporal coordinates. Usually one considers the central potential $[V_0(r), 0]$ approximation with the stationary solution:

$$\Psi(x) = \exp(-iEt/\hbar)\,\varphi(x), \qquad (2)$$

where $\varphi(x)$ is the solution of the stationary equation:

$$\{\frac{1}{c^2}[E+eV_0(r)]^2+\hbar^2\nabla^2-m^2c^2\}\varphi(x)=0 \qquad (3)$$

Here E is the total energy of the system (sum of the mass energy mc^2 and binding energy ε_0). In principle, the central potential V_0 naturally includes the central Coulomb potential, the vacuum-polarization potential, the strong interaction potential.

The most direct approach to treating the strong interaction is provided by the well known optical potential model (c.g. [2]). Practically in all papers the central potential V_0 is the sum of the following potentials. The nuclear potential for the spherically symmetric density $\rho(r|R)$ is [6,13]:

$$V_{nucl}(r|R) = -((1/r)\int_0^r dr'r'^2\rho(r'|R) + \int_r^\infty dr'r'\rho(r'|R) \qquad (4)$$

The most popular Fermi-model approximation the charge distribution in the nucleus $\rho(r)$ (c.f.[11]) is as follows:

$$\rho(r) = \rho_0/\{1+\exp[(r-c)/a)]\}, \qquad (5)$$

where the parameter $a=0.523$ fm, the parameter c is chosen by such a way that it is true the following condition for average-squared radius:

$$<r^2>^{1/2}=(0.836\cdot A^{1/3}+0.5700)\text{fm}.$$

The effective algorithm for its definition is used in refs. [12] and reduced to solution of the following system of the differential equations:

$$V'nucl(r,R) = (1/r^2)\int_0^r dr'r'^2\rho(r',R) \equiv (1/r^2)y(r,R)$$

$$, \qquad (6)$$

$$y'(r,R) = r^2\rho(r,R), \qquad (7)$$

$$\rho'(r) = (\rho_0/a)\exp[(r-c)/a]\{1+\exp[(r-c)/a)]\}^2 \qquad (8)$$

with the corresponding boundary conditions. Another, probably, more consistent approach is in using the relativistic mean-field (RMF) model, which been designed as a renormalizable meson-field theory for nuclear matter and finite nuclei [13].To take into account the radiation corrections, namely, the effect of the vacuum polarization we have used the generalized Ueling-Serber potential with modification to take into account the high-order radiative corrections [5,12].

The most difficult aspect is an adequate account for the strong interaction. In the pion-nucleon state interaction one should use the following pulse approximation expression for scattering amplitude of a pion on the "i" nucleon [2,3]:

$$f_i(r) = \{b'_0 + b'_1(t\tau) + [c'_0 + c'_1(t\tau)]kk'\}\delta(r-r_i); \qquad (9)$$

where t and τ are the isospines of pion and nucleon. The nucleon spin proportional terms of the kind $\sigma[kk']$ are omitted. The constants in (9) can be expressed through usual s-wave (α_{2T}) and p-wave $(\alpha_{2T,2J})$ scattering length (T and J -isospin and spin of he system πN. The corresponding parameters in the Compton wave length $\lambda\!\!\!^{-}_\pi$ terms are as follows:

$$b_0' = (\alpha_1 + 2\alpha_3)/3 = -0.0017\,\lambda_\pi .$$

$$b_0' = (\alpha_3 - \alpha_1)/3 = -0.086\,\lambda_\pi .$$

$$c_0' = (4\alpha_{33} + 2\alpha_{13} + 2\alpha_{31} + \alpha_{11})/3 = -0.208(\lambda_\pi)^3 .$$

$$c_1' = (2\alpha_{33} - 2\alpha_{13} + \alpha_{31} - \alpha_{11})/3 = -0.184(\lambda_\pi)^3 . \qquad (10)$$

The scattering amplitude for pion on a nucleus can be further received as a coherent sum of the πN-scattering lengths. рассеяния. In approximation of the only s-wave interaction the corresponding potential can be written in the Dezer form:

$$V_N(r) = -2\pi\hbar^2\mu_\pi^{-1}\left[ZA^{-1}a_p + (A-Z)A^{-1}a_n\right]\rho(r). \qquad (11)$$

The s-wave lengths of the $\pi^{-1}p$-scattering $a_p = (2\alpha_1 + \alpha_3)/3$ и $\pi^{-1}n$- scattering $a_n = \alpha_3$; scattering are introduced to Eq. (11). Because of the equality between $a_n = b_0' + b_1'$ and $a_p = b_0' - b_1'$ (with an opposite sign) the theoretical shift of the s-level with $T = 0\ (A = 2Z)$ from Eq. (12) is much less than the observed shift. So, the more correct approximation must take into account the effects of the higher orders.

In whole the energy of the hadronic atom is represented as the sum:

$$E \approx E_{KG} + E_{FS} + E_{VP} + E_N ; \qquad (12)$$

Here E_{KG} -is the energy of a pion in a nucleus (Z, A) with the point-like charge (dominative contribution in (12)), E_{FS} is the contribution due to the nucleus finite size effect, E_{VP} is the radiation correction due to the vacuum-polarization effect, E_N is the energy shift due to the strong interaction V_N.

The strong pion-nucleus interaction contribution can be found from the solution of the Klein-Gordon equation with the corresponding pion-nucleon potential.

3. Results and conclusions

In table 1 the data on the transition energies in some pionic atoms of ^{175}Lu, ^{205}Tl, ^{202}Pb (from. Refs. [4-7]): the measured values form the Berkley, CERN and Virginia laboratories, the theoretical values for the *4f-3d, 5g-4f* pionic transitions (E_{th1}^N - values from the Klein-Gordon-Fock equation with the pion-nucleus potential [2]; E_{KGF}- values from the Klein-Gordon-Fock equation with account of radiative corrections (our data); E_{KS} – the RMF finite nuclear size contribution (our data), E_{th2}^N - values from the Klein-Gordon-Fock equation with the generalized pion-nuclear potential [5] (our data).

The analysis of the presented data indicate on the necessity of the further more exact experimental investigations and further improvement of the pion-nuclear potential modelling.

Table 1

Transition energies (keV) in the spectra of some heavy pionic atoms (see text)

Atom	E_{EXP} Berkley	E_{EXP} CERN	E_{KGF}	E_{FS}	E_{th1}^N	E_{th2}^N
Transition *4f-3d*						
^{133}Cs	560,5 ± 1,1	562,0 ± 1,5	556,80	-0,33	561,47	560.88
^{205}Tl	-	-	-	963.920	-	968.25
Transition 5g-4f						
^{175}Lu	-	-	-	427.313	-	428.80
^{205}Tl	-	561.67 ± 0.25	559.65	559.681	560.93	561.63

One can see that the contributions provided by the finite size effect should be accounted in a precise theory. Really, under availability of the ”exact” values of the transitions energies one can perform the comparison of the theoretically and experimentally defined transition energies in the X-ray spectra in order to make a redefinition of the pion-nucleon model potential parameters using Eqs. (9)-(11). Taking into account the increasing accuracy of the X-ray pionic atom spectroscopy experiments, one can conclude that the such a way will make more clear the true values for parameters of the pion-nuclear potentials and correct the disadvantage of widely used parameterization of the potentials (9)-(11).

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Abstract

New relativistic method of the Klein-Gordon-Fock equation with an generalized pion-nuclear potential is used to determine the transition energies with accounting for the strong pion-nuclear interactions effects in spectroscopy of some heavy pionic atoms. As example, there is carried out studying the Coulomb, nuclear and strong interaction contributions into the 4f-3d, 5g-4f transitions energies for the ^{175}Lu, ^{205}Tl, ^{202}Pb pionic atoms.

Key words: strong interaction, pionic atom, relativistic theory

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РЕЛЯТИВИСТСКАЯ ТЕОРИЯ СПЕКТРОВ ТЯЖЕЛЫХ ПИОННЫХ АТОМОВ С УЧЕТОМ ЭФФЕКТОВ СИЛЬНОГО ПИОН-ЯДЕРНОГО ВЗАИМОДЕЙСТВИЯ: НОВЫЕ ДАННЫЕ ДЛЯ ^{175}Lu, ^{205}Tl, ^{202}Pb

Резюме

Новый релятивистский метод на основе уравнения Клейна-Гордона-Фока с обобщенным пион-ядерным потенциалом применен к вычислению энергий переходов с учетом эффектов сильного пион-ядерного взаимодействия в спектроскопии некоторых тяжелых пионных атомов. В качестве примера проведено детальное изучение кулоновского, ядерного вкладов, вклада за счет сильного взаимодействия в энергии переходов 4f-3d, 5g-4f для ^{175}Lu, ^{205}Tl, ^{202}Pb пионных атомов.

Ключевые слова: сильное взаимодействие, пионный атом, релятивистская теория

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РЕЛЯТИВІСТСЬКА ТЕОРІЯ СПЕКТРІВ ВАЖКИХ ПІОННИХ АТОМІВ З УРАХУВАННЯМ ЕФЕКТІВ СИЛЬНОЇ ПІОН-ЯДЕРНОЇ ВЗАЄМОДІЇ: НОВІ ДАННІ ДЛЯ ^{175}Lu, ^{205}Tl, ^{202}Pb

Резюме

Новий релятивістський метод на основі рівняння Клейна-Гордона-Фока із узагальненим піон-ядерним потенціалом застосовано до розрахунку енергій переходів з урахуванням ефектів сильної піон-ядерної взаємодії в спектроскопії декотрих важких піонних атомів. В якості прикладa проведено докладне вивчення кулонівського, ядерного внесків, внеску за рахунок сильної взаємодії в енергії переходів 4f-3d, 5g-4f для ^{175}Lu, ^{205}Tl, ^{202}Pb піонних атомів.

Ключові слова: сильна взаємодія, піонний атом, релятивістська теорія