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SIMULATION CHAOTIC DYNAMICS OF RELATIVISTIC BACKWARD-WAVE OSCILLATOR WITH USING CHAOS THEORY AND QUANTUM NEURAL NETWORKS

Nonlinear simulation and forecasting chaotic evolutionary dynamics of such complex system as relativistic backward-wave oscillator is treated using the new combined method, based on the chaos theory algorithms, concept of geometric attractors, and algorithms for quantum neural network simulation. It has been performed modelling the dynamics of multilayer photon echo neural network for the case of noisy input sequence. It has been performed analysis, modelling and processing the temporal dependence of the output amplitude for the backward-wave oscillator, described by system of the nonstationary nonlinear theory equations for the amplitude of electromagnetic field and motion of a beam. The data on the Lyapunov's exponents, Kolmogorov entropy, correlation coefficient between the actual and neural networks prognostic rows, referred to a temporal dependence of the output signal amplitude of the nonrelativistic (relativistic) backward-wave oscillator are listed. The combining the advanced algorithms of the modern chaos theory, concept of a compact geometric attractors and one of the effective neural network algorithms, or, in a more general using an effective model of artificial intelligence etc, could provide very adequate and quantitatively correct description of temporal evolutionary dynamics of most complicated systems, in particular, in the field of modern ultrahigh-frequency electronics

1. Introduction.

It is well known that the multiple physical, chemical, biological, technical, communication, economical, geophysical and other systems (devices) demonstrate the typical complex chaotic behaviour. In many important situations typical dynamics of these systems is a world of strong nonlinearity. Naturally, there is a quite considerable number of works, devoted to an analysis, modelling and prediction evolutionary dynamics of different complex systems from the viewpoint of theory of dynamical systems and chaos, fractal sets of physics and other systems [1-11]. In a series of papers [10-20] the authors have attempted to apply some of these methods in a variety of the physical, geophysical, hydrodynamic problems. In connection with this, there is an extremely important task on development of new, more effective approaches to the nonlinear modelling and prediction of chaotic processes in different complex systems (e.g.[1-22]). Especial interest attracts research of regular and chaotic dynamics of nonlinear processes in various classes of devices of the so-called relativistic ultrahigh-frequency electronics (see, e.g. [1-8]). Earlier we have developed an effective approach to analysis, modelling and forecasting chaotic evolutionary dynamics of complex systems , which is based on the based on the chaos theory algorithms, concept of geometric attractors, and algorithms for quantum neural network simulation [17-20]. We are developing a new approach to analyze complex system dynamics based on the chaos theory methods and neural network algorithms [21-27]. The basic idea of the construction of the cited approach to prediction of chaotic processes in complex systems is in the use of the traditional concept of a compact geometric attractor in which evolves the fundamental dynamic characteristics, plus the implementation of neural network algorithms. The existing so far in the theory of chaos prediction models are based on the concept of an attractor, and are described in a number of papers (e.g. [17-24]). For example, very useful review of the subfield that has attracted great interest in the last years, namely, the application of various approaches from complex network theory in the context of nonlinear time series analysis, has been presented in ref. [21,26] (see refs. therein). The time series mining focuses more on indexing, clustering, classification, segmentation, discovery, and forecasting [21]. According to [21], there has been a considerable amount of rapid developments of data mining tools initiated by the advent of big data and cloud computing reflecting the increasing size and complexity of available datasets. It should be underlined that hitherto however, there has been practically no multiple overlap between the nonlinear time series analysis and complex neural network methods (e.g. [13-15]).

In this paper we present the results of the nonlinear simulation and forecasting chaotic evolutionary dynamics of such complex system as relativistic backward-wave oscillator [5-7] using earlier developed new combined method [22-27], based on the chaos theory algorithms, concept of geometric attractors, and algorithms for quantum neural network simulation.

2. Mathematical approach

As the main fundamental ideas of the our combined approach, based on the chaos theory algorithms, concept of geometric attractors, and algorithms for quantum neural network simulation [17-20], here we will concern only the principally important items of this studying. As usually, let us remind that from a mathematical point of view, it is a fact that in the phase space of the system an orbit continuously rolled on itself due to the action of dissipative forces and the nonlinear part of the dynamics, so it is possible to stay in the neighbourhood of any point of the orbit *y (n)* other points of the orbit $y^r(n)$, $r = 1,2,..., N_B$, which come in the neighbourhood *y (n)* in a completely different times than *n*.

According to ref. [22-24,27], in terms of the modern theory of neural systems, and neuro-informatics (e.g. [12,15]), the process of modelling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations (miemo-dynamic equations). Considering the neural network with a certain number of neurons, as usual, we can introduce the operators S_{ij} synaptic neuron to neuron u_i u_j, while the corresponding synaptic matrix is reduced to a numerical matrix strength of synaptic connections: $W = || w_{ij} ||$. The operator is described by the standard activation neuro-equation determining the evolution of a neural network in time:

$$s_i' = sign(\sum_{j=1}^{N} w_{ij} s_j - \theta_i), \qquad (1)$$

where 1<i<N. Naturally it easily to understand that a state of the neuron (the chaos-geometric interpretation of the forces of synaptic interactions, etc.) can be represented by currents in the phase space of the system and its the topological structure is obviously determined by the number and position of attractors. To determine the asymptotic behavior of the system it becomes crucial an information aspect of the problem, namely, the fact of being the initial state to the basin of attraction of a particular attractor. The domain of attraction of attractors are separated by separatrices or certain surfaces in the phase space. Their structure, of course, is quite complex, but mimics the chaotic properties of the studied object. Then, as usual, the next step is a natural construction parameterized nonlinear function ***F** (x, **a**)*, which transforms:

$$\mathbf{y}(n \rightarrow \mathbf{y}(n+1) = \mathbf{F}(\mathbf{y}(n), \mathbf{a}), \qquad (2)$$

and then to use the different (including neural network) criteria for determining the parameters a (see below). Although, according to a classical theorem by Kolmogorov-Arnold -Moser, the dynamics evolves in a multidimensional space, the size and the structure of which is predetermined by the initial conditions, this, however, does not indicate a functional choice of model elements in full compliance with the source of random data. One of the most common forms of the local model is the model of the Schreiber type [3] (see also [17-20]).

The easiest way to implement this program is in considering the original local neighbourhood, enter the model(s) of the process occurring in the neighbourhood, at the neighbourhood and by combining together these local models, designing on a global nonlinear model. The latter describes most of the structure of the attractor. As shown

Schreiber [3], the most common form of the local model is very simple:

$$s(n+\Delta n)=a_0^{(n)}+\sum_{j=1}^{d_A} a_j^{(n)} s\left(n-(j-1)\tau\right) \quad (3)$$

where Δn is the time period for which a forecast. The coefficients $a_j^{(k)}$, may be determined by a least-squares procedure, involving only points $s(k)$ within a small neighbourhood around the reference point. Thus, the coefficients will vary throughout phase space. The fit procedure amounts to solving $(d_A + 1)$ linear equations for the $(d_A + 1)$ unknowns. When fitting the parameters a, several problems are encountered that seem purely technical in the first place but are related to the nonlinear properties of the system. If the system is low-dimensional, the data that can be used for fitting will locally not span all the available dimensions but only a subspace, typically. Therefore, the linear system of equations to be solved for the fit will be ill conditioned. However, in the presence of noise the equations are not formally ill-conditioned but still the part of the solution that relates the noise directions to the future point is meaningless. Other details of modelling techniques are described, for example, in refs. [22-26].

The new element of our approach is using the NNW algorithm in forecasting nonlinear dynamics of chaotic systems [9,10]. In terms of the neuro-informatics and neural networks theory the process of modelling the evolution of the system can be generalized to describe some evolutionary dynamic neuro-equations. Imitating the further evolution of a system within NNW simulation with the corresponding elements of the self-study, self-adaptation, etc., it becomes possible to significantly improve the prediction of its evolutionary dynamics. The fundamental parameters to be computed are the Kolmogorov entropy (and correspondingly the predictability measure as it can be estimated by the Kolmogorov entropy), the LE, the KYD etc. The LE are usually defined as asymptotic average rates and they are related to the eigenvalues of the linearized dynamics across the attractor. Naturally, the knowledge of the whole LE allows to determine other important invariants such as the Kolmogorov entropy and the attractor's dimension. The Kolmogorov entropy is determined by the sum of the positive LE. The estimate of the dimension of the attractor is provided by the Kaplan and Yorke conjecture $d_L = j + \sum_{i=1}^{j} \lambda_i / |\lambda_{j+1}|$, where j is such that $\sum_{i=1}^{j} \lambda_i > 0$ and $\sum_{i=1}^{j+1} \lambda_i < 0$, and the LE are taken in descending order.

3. Standard dynamical model of a backward-wave oscillator and neural networks modelling.

Nonlinear dynamics of the system (the backward-wave oscillator) is usually described by system of the nonstationary nonlinear theory equations for the evolution in time and space for the amplitude of the electromagnetic field and the motion of the beam (single model BWT) (e.g.[7,8]):

$$\frac{\partial F}{\partial \tau}-\frac{\partial F}{\partial \xi}=-\frac{\mathscr{L}}{\pi}\int_0^{2\pi} e^{-i\theta_\alpha}\,d\alpha, \quad (4)$$

$$\frac{\partial^2\theta}{\partial \xi^2}=-\mathscr{L}^2\,\mathrm{Re}\,F e^{i\theta_\alpha}$$

with the corresponding boundary and initial conditions:

$$\theta_\alpha|_{\xi=0}=\alpha, \qquad \left.\frac{\partial \theta_\alpha}{\partial \xi}\right|_{\xi=0}=0, \quad (5)$$

$$F|_{\xi=1}=0, \qquad F|_{\tau=0}=F(\xi,0),$$

Here $\theta(\xi, \tau, \theta_0)$ is a phase of the electron, which runs in a space of interaction with phase θ_0 in a field, $F(\xi,\tau)$ dimensionless complex amplitude of the wave $E(x,t)=\mathrm{Re}[E(x,t)\exp(i\omega_0 t - i\beta_0 x)]$, $\xi = \beta_0 C x$ – the dimensionless coordinate, $L = \beta_0 l C = 2\pi C N$- the dimensionless length of the interaction space, l is a length of a system, N-

is a number of slow waves, covering over the length of system, $C = \sqrt[3]{I_0 K_0/(4U)}$ is the known Pierce parameter , I_0 is a current of beam, U is an accelerated voltage, K_0-resistance of link of the slowing system,

$$\tau = \omega_0 C\,(t - x/v_0)\,(1 + v_0/v_{\text{гр}})^{-1}$$

(i.e. $\tau\tau - \xi / v_0$) is the dimensionless "retarded" time, C is modified gain parameter (see more details in refs. [2-8]). In refs. [5-8] there is presented the detailed information about numerical solution of the corresponding system and performed an analysis of the fundamental topological and dynamical invariants. For numerical simulation we have used a software package, based on the photon echo neural network, which imitates evolutionary dynamics of the complex system [15]. It has the following key features: multi-layering, possibility of introducing training, feedback and controlled noise. There are possible the different variants of the connections matrix determination and binary or continuous sigmoid response (and so on) of the model neurons. In order to imitate a tuition process we have carried out numerical simulation of the neural networks for recognizing a series of patterns (number of layers N=5, number of images p=640; the error function:

$$SSE = \sum_{p=1}^{p_{\max}} \{..\sum_{k=}^{k_{\max}} [t(p,k) - O(p,k)]^2\}, \qquad (6)$$

where $O(p,k)$ – neural networks output k for image p and $t(p,k)$ is the trained image p for output κ; SSE is determined from a procedure of minimization; the output error is $RMS=sqrt(SSE/P_{max})$; As neuronal function there is used function of the form: $f(x) = 1/[1+\exp(-\delta x)]$. In our calculation there is tested the function $f(x,T)=\exp[(xT)^4]$ too.

In order to check the possibilities of the (neural networks package NNW-13-2003 [15]) of the multilayer neural networks, it has been performed processing noisy input sequence. Fig. 2 demonstrates the results of modeling the dynamics of multilayer neural network for the case of noisy input sequence [24]. The input signal was the Gaussian-like pulse with adding a noise with intensity D. At a certain value of the parameter D (the variation interval .0001-0.0040) the network training process and signal playback is optimal. The optimal value of D is 0.00168.

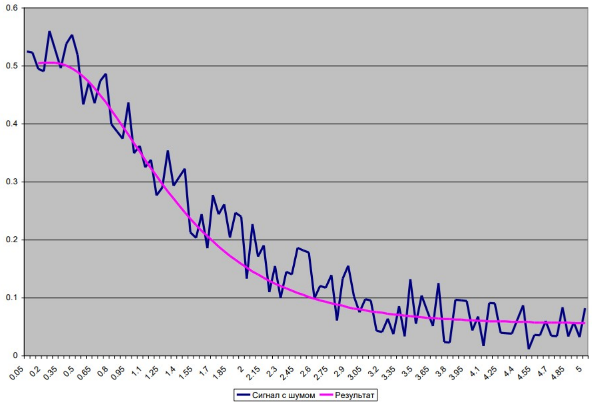

Figure 1. The results of modeling the dynamics of multilayer neural for the case of noisy input sequence (see text).

The farther step was in analysis, modelling and processing the temporal dependence of the out amplitude (solution of the system (4)-(5). For illustration we present the results of calculation the Lyapunov's exponents, the Kolmogorov entropy K_{entr} for the system, which is described by the system (4)-(5). The governing parameter L is equal 4.05.

Table 1. Numerical simulation data for the Lyapunov exponents (LE: λ_1–λ_4) and the Kolmogorov entropy K (L=4.05)

λ_1	λ_2	λ_3	λ_4	K
0.261	0.0001	–0.0004	–0.528	0.26

In table 2 we present data on the correlation coefficient (r) between the actual and neural networks prognostic rows, referred to the temporal dependence of the output signal amplitude of the backward-wave oscillator (1) for L=4.05 (NN is number of neighbours for pre-ahead 100 points numerical series of the amplitude temporal dependence). The more details can be found in ref. [6,22-24]. One can see very closed coincidence between the actual and predicted row values for the temporal dependence of the output signal amplitude of the backward-wave oscillator (tube).

Table 2. The correlation coefficient (r) between the actual and prognostic rows, referred to the temporal dependence of the output signal amplitude of the backward-wave oscillator (1) for L=4.05 (see text) .

NN	85	225	250
r	0.94	0.96	0.96

Analysis of the PC experiment results allows to make conclusion about sufficiently high-quality processing the input signals of very different shapes and complexity by the applied neural network. This is concerning the results of modelling the temporal dependence of the output signal amplitude of the backward-wave oscillator (1). We believe that the combining the advanced algorithms of the modern chaos theory, concept of a compact geometric attractors and one of the effective neural network algorithms, or, in a more general using an effective model of artificial intelligence etc, could provide very adequate, quantitatively correct description of temporal evolutionary dynamics of most complicated systems, in particular, in the field of modern ultrahigh-frequency electronics (see, e.g. [1-8]).

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Summary. Nonlinear simulation and forecasting chaotic evolutionary dynamics of such complex system as nonrelativistic (relativistic) backward-wave oscillator is treated using the new combined method, based on the chaos theory algorithms, concept of geometric attractors, and algorithms for quantum neural network simulation. It has been performed modeling the dynamics of multilayer photon echo neural network for the case of noisy input sequence. It has been performed analysis, modelling and processing the temporal dependence of the out amplitude for the backward-wave oscillator, described by system of the nonstationary nonlinear theory equations for the amplitude of the electromagnetic field and the motion of the beam. The data on the Lyapunov's exponents, Kolmogorov entropy and the correlation coefficient between the actual and neural networks prognostic rows, referred to the temporal dependence of output signal amplitude of the nonrelativistic (relativistic) backward-wave oscillator (tube) are listed. The combining the advanced algorithms of the modern chaos theory, concept of a compact geometric attractors and one of the effective neural network algorithms, or, in a more general using an effective model of artificial intelligence etc, could provide very adequate and quantitatively correct description of temporal evolutionary dynamics of most complicated systems, in particular, in the field of modern ultrahigh-frequency electronics.

Key words: Keywords: non-relativistic and relativistic backward-wave oscillator (tube), spectrum of radiation, spectroscopy, chaotic dynamics, concept of geometric attractor. quantum neural networks

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МОДЕЛЮВАННЯ ХАОТИЧНОЇ ДИНАМІКИ РЕЛЯТИВІСТСЬКОЇ ЛАМПИ ОБЕРНЕНОЇ ХВИЛІ З ВИКОРИСТАННЯМ ТЕОРІЇ ХАОСУ ТА КВАНТОВИХ НЕЙРОМЕРЕЖ

Резюме. Нелінійне моделювання та прогнозування хаотичної еволюційної динаміки такої складної системи, як лампа зворотної хвилі, розглядається за допомогою нового комбінованого методу, заснованого на алгоритмах теорії хаосу, концепції геометричних атракторів та квантово нейронно-мережевих алгоритмах моделювання. Проведено моделювання динаміки багатошарової фотонної нейромережі для випадку зашумленої вхідної послідовності. Проведено аналіз, моделювання та обробка часової залежності вихідної амплітуди для нерелятивістської (релятивістської) лампи, динаміка якої описується системою рівнянь нестаціонарної нелінійної теорії для амплітуди електромагнітного поля та руху пучка. Наведені дані про показники Ляпунова, ентропію Колмогорова, коефіцієнт кореляції між фактичним і нейронно-мережевими прогностичними даними часової залежності амплітуди вихідного сигналу розглянутої лампи зворотної хвилі тощо. Поєднання удосконалених алгоритмів сучасної теорії хаосу, концепції компактних геометричних атракторів і одного з ефективних нейронно-мережевих алгоритмів, або, у більш загальному сенсі, використання ефективної моделі штучного інтелекту тощо, може забезпечити дуже адекватний і кількісно коректний опис часової еволюційної динаміки найскладніших фізичних систем, зокрема, у сфері сучасної надвисокочастотної електроніки.

Ключові слова: нерелятивістська та релятивістська лампа зворотної хвилі, спектр випромінювання, спектроскопія, хаотична динаміка, концепція геометричного атрактору, квантові нейронні мережі

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