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NON-LINEAR ANALYSIS OF CHAOTIC SELF-OSCILLATIONS IN BACKWARD-WAVE TUBE

The paper presents the results of the analysis and modelling of topological and dynamic invariants for the regimes of chaotic self-oscillations in the backward-wave oscillator, in particular, the analysis of chaotic time series for the amplitude of the output signal, which is the solution of the equations of the non-stationary nonlinear theory for the O-type backward-wave oscillator (without taking into account space charge, relativistic effects, energy losses, etc.). The main attention is paid to the calculation and analysis of the spectrum of Lyapunov exponents based on the Sano-Savada algorithm. Numerical data of the Lyapunov backward-wave oscillator calculated for the time series of the amplitudes of the output signal are given, which definitely indicate the presence of elements of advanced chaos in the dynamics of the system.

1. At present time study the powerful generators of chaotic oscillations of microwave range are of a great interest for radar, plasma heating in fusion devices, modern systems of information transmission using dynamic chaos and many other applications. Among the most studied of vacuum electronic devices with complex dynamics are backward-wave oscillators, for which the possibility of generating chaotic oscillations has been theoretically and experimentally found [1-12]. The backward-wave oscillator is an electronic device for generating electromagnetic vibrations of the superhigh frequencies range. Above nost interesting for our study papers one should pay the attention on the following works.

Firstly, the papers [4,5] contain very important results. The authors of these works have solved the equations of nonstationary nonlinear theory for the O-type backward-wave oscillator without account of the spatial charge, relativistic effects, energy losses etc. It has been shown that the finite-dimension strange attractor is responsible for chaotic regimes in the backward-wave oscillator. The authors have in details presented the numerical data on the different dynamical characteristics of the non-relativistic backward-wave oscillator, namely, phase portraits, statistical quantifiers for a weak chaos arising via period-doubling cascade of self-modulation and the same characteristics of two non-relativistic backward-wave oscillator. It is shown that the chaos formed in the dynamics of the oscillator is characterized by more than one positive Lyapunov exponent (developed chaos or hyperchaos). the Lyapunov correlation dimensions as well as in general a whole set of the dynamical and topological characteristics of the strange attractor are also calculated.

Naturally the quantitative study of the dynamical and topological characteristics of the strange attractors in dynamics of the nonrelativistic and relativistic backward-wave oscillators have been performed in many papers (e.g. [1-12] and refs. therein) Depending on the key control parameters of the system, many researchers have discovered a rather complex scheme of alternating regular, auto-modulation, and chaotic (hyperchaotic) generation modes, and in the end, the processes ended with a transition to highly irregular broadband chaotic (hyperchaotic) oscillations with a sufficient spectrum.

In the last decade, a large number of numerical studies have been carried out using ideas derived from the science of chaos to characterize, model and predict the regular and chaotic dynamics of various electronics systems, including the studies of the dynamics of backward wave oscillators listed above (see [1-6,24]). The results of such studies are very encouraging, since they not only showed that the dynamics of complex and regular and chaotic phenomena can be understood with a sufficiently high accuracy from the quantitative point of view, but also proved sufficiently efficient models for predicting the dynamics (time series) of complex chaotic systems, at least in the first approximation (e.g.[7-22]).

A variety of different techniques for characterizing chaotic dynamics of the nonlinear systems identifying the presence of chaotic elements is used [1,2]. Usually a mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's analysis, and surrogate data method and others are used for comprehensive characterization. The fundamental quantities to characterize chaotic behaviour of the complex dynamical systems are the exponential divergence of nearby orbits (computing positive Lyapunov exponents'), positive finite Kolmogorov entropy, and a noninteger dimension of the attractor [7-26]. These characteristics are usually invariant under the corresponding smooth transformation of coordinates. There are several determination schemes among these quantities, and if the Lyapunov spectrum can be determined, the rest can be estimated as equalities or upper or lower bounds. So, the definition of the highest Lyapunov exponents, as well as the full spectrum of the Lyapunov exponents is an important task in the nonlinear analysis of the complex dynamical distributed. It is well known that the Lyapunov exponents are a quantitative measure sensitivity to the values of the initial conditions. Since the Lyapunov exponents are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor There are several different methods for computing the Lyapunov exponents [e.g.2,3,13-25]). One of the most spread algorithms to compute the leading Lyapunov exponents is Benettin algorithm [21] (see also [4,5]. The generalized algorithm allows to compute the full spectrum of Lyapunov. The disadvantage of the algorithm is that its applicability is only in those cases where when the corresponding evolutionary equations of the studied dynamical system known; besides, it is possible to measure all of it phase coordinates, which is not always possible.

Another approach is in computing the senior indicator for a sample from a single coordinate. The algorithm named after Wolf et al [20] calculates the leading Lyapunov exponent from sampled from a single coordinate, and is used when the equations are unknown evolution of the system. This algorithm gives quite satisfactory results, but requires very large samples, which is problematic for real data sets. In the last years it has become very popular computing the full Lyapunov spectrum using neural networks algorithms (e.g. [3,7-11,23-26] and reds therein). The use and optimization of neural networks is one of the directions in improving the performance and accuracy of algorithms for calculating Lyapunov exponents. Besides, it should be noted that in the latter it could be possible to organize a training a neural network on samples, which allows use short samples or noisy data. This gives significant advantages over classical methods. At last , it is necessary to add that in order to increase an accuracy of computing the Lyapunov exponents, one should use various orthogonalization methods , for example, the classical Gram-Schmidt method and the modified Gram-Schmidt method.

This paper is devoted to numerical studying the chaotic self-oscillations regimes in the backward-wave tube, namely, to application of the some numerical analysis techniques to analysis of the chaotic time series as solutions of the equations of nonstationary nonlinear theory for the O-type backward-wave oscillator. The main attention is paid to computing and analysing a spectrum of the Lyapunov exponent’s within the Sano-Sawada method [23] (see other details in refs. [2,9,10]). The advanced numerical data on the Lyapunov exponents of computed for the time series of output signal amplitudes, which are the solutions of nonstationary nonlinear theory for the O-type backward-wave oscillator without account of the spatial charge, relativistic effects, energy losses etc.

2. The main mathematical object of the numerical investigation is the time or other series of the amplitude of the dynamic parameter of the system, which is to be carried out, we introduce it formally:

$$x(t_0 + n\Delta t) = x(n), \quad (1)$$

where t_0 – some initial point in time, Δt is the time interval during which further members of the time series are selected, $t_0 + n\Delta t$ – "i" point in time.

By definition, the procedure of construction (restoration) from a scalar time series { s_i } to a series of state vectors { y_i } is called phase trajectory reconstruction. It should be noted that depending on the class of the problem, more precisely, the system or device under investigation, any dynamic parameters can act as y(n),

Further it is usually necessary to reconstruct phase space using as well as possible information contained in the dynamical parameter s(n), where n the number of the measurements. Such a reconstruction results in a certain set of d-dimensional vectors y(n) replacing the scalar measurements. Packard et al. [7] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $x(n + \tau)$, where τ is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. Then using a collection of time lags to create a vector **x** in d dimensions,

$$\mathbf{x}(n) = [x(n), x(n + \tau), x(n + 2\tau), \ldots, x(n + (d-1)\tau)], \quad (2)$$

the required coordinates are provided.

In a nonlinear system, the $\mathbf{x}(n + j\tau)$ are some unknown nonlinear combination of the actual physical variables that comprise the source of the measurements. The dimension d is called the embedding dimension, dE. If τ is chosen too small, then the coordinates $x(n + j\tau)$ and $x(n + (j + 1)\tau)$ are so close to each other in numerical value that they cannot be distinguished from each other. Similarly, if τ is too large, then $x(n + j\tau)$ and $x(n + (j + 1)\tau)$ are completely independent of each other in a statistical sense. Also, if τ is too small or too large, then the correlation dimension of attractor can be under- or overestimated respectively [8,18]. The autocorrelation function and average mutual information can be applied here.

Usually the main aim of the embedding dimension determination is to reconstruct a Euclidean space R_d large enough so that the set of points dA can be unfolded without ambiguity. There are several standard approaches to reconstruct the attractor dimension (see, e.g., [2,3,11-17]). The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, C(r), to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [16] is the most commonly used approach.

The Lyapunov exponents are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local Lyapunov exponents. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion.

In our case the quantity If $\{x_n)$ (n=1,2, . . .) denote a time series of some physical quantity measured at the discrete time interval (0 in fact this is an output signal amplitudes, which are the solutions of nonstationary nonlinear theory for the O-type backward-wave oscillator).

Further, according to the the Sana-Sawada algorithm [23], one should consider a small ball of radius ε centered at the orbital point x,, and find any set of points $\{x_i)$ (i =1,2, . . . , N) included in this ball, i.e.:

$$\{y_i) = \{|x_{ki} — x_i| \; || \; x_k — x_i \; || < \varepsilon,\}, \quad (3)$$

where y_i is the displacement vector between x_k and x_i. After the evolution of a time interval $\tau = m\Delta t$ t, the orbital point x_j will proceed to x_{j+m} and neighboring points $\{x_k\}$to

$\{x_{k+m}\}$. The displacement vector y'=x,,—x is thereby mapped to

$$\{z_i)=\{|x_{ki}—x_i| \, || \, x_k—x_i \, || < \varepsilon,\}, \qquad (4)$$

According to Ref. [23], if the radius ε is small enough for the displacement vectors {y} and {z}to be regarded as good approximation of tangent vectors in the tangent space, evolution of y' to z' can be represented by some matrix A, as: $z^i=A_j\, y^i$.

Further one should proceed proceed to the optimal estimation of the linearized flow map A; from the data sets {y} and {z}. According to Ref. [23], a plausible procedure for optimal estimation is the least-square-error algorithm, which minimizes the average of the squared error norm between z and Ay with respect to all components of the matrix A is as follows:

$$\min \bar{S}\,(A_J) = \min\left(\frac{1}{N}\right)\sum_{m=1}^{N} \|zi \; - Ajyi\| ** 2$$

The Lyapunov exponents can be computed in a standard way as the corresponding limit of a sum $ln||Ajej||$. Other details can be found in Refs. [23,24]. In fact, if one manages to derive the whole spectrum of Lyapunov exponents, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, K, measures the average rate at which information about the state is lost with time. An estimate of this measure is the sum of the positive Lyapunov exponents. The inverse of the Kolmogorov entropy is equal to the average predictability.

Let us present some obtained results. All input data have been taken from ref. [4,5,24].

In Table 1 we present our data on the correlation dimension d_2, the embedding dimension determined based on the algorithm of false nearest neighboring points (d_N) with percentage of false neighbors (%) calculated for different values of time lag ▲ for two regimes of of chaotic self-oscillations in the backward-wave oscillator, in particular, for the amplitude of the output signal for L=4.1 (I) and L=6.1 (II). Here L is normalized backward-wave oscillator length , which plays a role of the main control (governing) parameter of the backward-wave oscillator (e.g.[4,5]). The corresponding amplitude of the output signal is the solution of the equations of the non-stationary nonlinear theory for the O-type backward-wave oscillator (without taking into account space charge, energy losses, etc.) (e.g.[4,24]).

Table 1. Correlation dimension d_2, the dimension of the attachment determined based on the algorithm of false nearest neighboring points (d_N) with percentage of false neighbors (%) calculated for different values of time lag τ

Chaos (I)			Hyperchaos (II)		
τ	d_2	(d_N)	τ	d_2	(d_N)
60	3.6	5 (5.5)	67	7.2	10 (12)
6	3.1	4 (1.1)	10	6.4	8 (2.1)
8	3.1	4 (1.1)	12	6.4	8 (2.1)

In Table 2 we list the results of computing the Lyapunov's exponents and Kolmogorov entropy K_{entr}.

Table 2. Numerical parameters of the chaotic self-oscillations in the backward-wave tube: λ_1–λ_4 are the Lyapunov exponents in descending order, *K* is Kolmogorov entropy

Regime	λ_1	λ_2	*K*
Chaos (L=4.2)	0.261	0.0001	0.26
Hyperchaos (L=6.1)	0.514	0.228	0.74
Regime	λ_3	λ_4	*K*
Chaos (L=4.2)	–0.0004	–0.528	0.26
Hyperchaos (L=6.1)	0.0000	–0.0002	0.74

One could see the that the Lyapunov exponents have as positive as negative values. So, the main conclusion is that the chaos formed in the dynamics of the oscillator is characterized by more than one positive Lyapunov exponent (developed chaos or hyperchaos). The received data are quite satisfactory agreed with the corresponding results of [4,24]. However, some difference in values of the Lyapunov exponents is connected with using another algorithm.

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Summary. The paper presents the results of the analysis and modelling of topological and dynamic invariants for the regime of chaotic self-oscillations in the backward-wave oscillator, in particular, the analysis of chaotic time series for the amplitude of the output signal, which is the solution of the equations of the non-stationary nonlinear theory for the O-type backward-wave oscillator (without taking into account space charge, relativistic effects, energy losses, etc.). The main attention is paid to the calculation and analysis of the spectrum of Lyapunov exponents based on the Sano-Savada algorithm. Numerical data of the Lyapunov backward-wave oscillator calculated for the time series of the amplitudes of the output signal are given, which definitely indicate the presence of elements of advanced chaos in the dynamics of the system.

Key words: non-relativistic and relativistic backward-wave tube, spectrum and dynamics, non-linear methods, optical chaos, Lyapunov exponents

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НЕЛІНІЙНИЙ АНАЛІЗ ХАОТИЧНИХ АВТОКОЛИВАЛЬНИХ РЕЖИМІВ У ЛАМПІ ЗВЕРНЕНОЇ ХВИЛІ

Резюме. В роботі представлені результати аналізу та моделювання топологічних і динамічних інваріантів для режиму хаотичних автоколивань в лампі зворотної хвилі, зокрема, виконаний аналіз хаотичних часових рядів для амплітуди вихідного сигналу, яка є розв’язком рівнянь нестаціонарної нелінійної теорії для лампи зворотної хвилі О-типу (без урахування просторового заряду, релятивістських ефектів, втрат енергії тощо). Основна увага приділена обчисленню та аналізу спектра показників Ляпунова на основі алгоритму Сано-Савади. Наведено чисельні дані показників Ляпунова розрахованих для часового ряду амплітуд вихідного сигналу, які безумовно вказують на наявність елементів розвиненого хаосу в динаміці системи.

Ключові слова: нерелятивістська та релятивістська лампа зворотної хвилі, спектр і динаміка, нелінійні методи, оптичний хаос, показники Ляпунова

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