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ON SOME NUMERICAL MODEL TO SOLVING DYNAMICAL EQUATIONS OF NONRELATIVISTIC AND RELATIVISTIC BACKWARD-WAVE TUBE

It is developed an effective computational approach to solution the master corresponding system of differential equations, which describe the nonlinear stationary and non-stationary electromagnetic processes in the nonrelativistic and relativistic backward-wave tubes (carcinotrons) with maximal accounting for the different physical factors such as the relativistic effects, effects of dissipation, the presence of a space charge, wave reflections at the ends of the slowing system, stochastic factors by means including the special elements in a whole system etc as well as the detailed investigation of characteristics (dynamical and topological invariants) of dynamics of a carcinotron in automodulation and chaotic regimes with construction the corresponding bi-furcation diagrams. Below in order to further solve the master system of dynamical equations for carcinotron it is presented in brief the realizing numerical scheme, based on the use of the conservative finite-difference schemes of the "predictor-corrector" type and the sweep algorithm.

1. Introduction

One of the most quickly developing directions of the modern physical, quantum, sensor and phototo-electronics is theoretical and experimental study of physical processes in systems and devices of relativistic microwave range electronics, including the backward-wave tubes (lamps, oscillators) their chains and others under different regimes of their functionating (for example, look [1-12]).

It is known that the backward-wave tubes or carcinotrons are an electronic devices for generating electromagnetic oscillations in the microwave range, in which a beam of electrons interacts with an electromagnetic wave in a retarding system in situations where the phase speed of the wave is close to the speed of electrons, and the group speed is opposite in direction [1–8]. Thanks to the first condition, the electrons are exposed to the effective action of the wave field: clumps are formed in the beam, and a high-frequency component of the current appears. Surely, at the present time there are developed a great number of different simple and quite complicates stationary and nonstationary models to describe the nonlinear electromagnetic processes in the nonrelativistic and relativistic backward-wave tubes (for example, look [1-8] and refs therein).

There have been presented quite much numerical approaches to find numerical solutions of the stationary and nonstationary systems of differential equations which are describe the nonlinear electromagnetic processes in the nonrelativistic and relativistic backward-wave tubes [3-8].

It should be stated that the implementation of mathematical models on computers takes place using the methods of applied mathematics, which, of course, are constantly being improved along with progress in the field of computer technology. The solution of the mathematical model of the problem, which should provide the criterion of efficiency and optimality, can be obtained faster with the help of a suitable efficient algorithm. Any reduction of the problem of relativistic microwave range (little and large power) electronics, including the backward-wave tubes, is usually reduced to the solution of algebraic equations of one or another structure (for example, look details in [9-16]).

As a result, most methods of applied mathematics are related to reducing the problem to a system of algebraic equations and their subsequent solution. One of the fairly widespread methods of solving problems in mathematical physics and applied mathematics is the finite difference method [15] (see also [16-20]).

In recent years, the problem of wide application of various methods of constructing difference schemes in the problems of mathematical physics, physics of systems, elements and devices, including physical and quantum electronics [3-12].

The main aim of our work is develop effective computational approach to solution the master corresponding system of differential equations, which describe the nonlinear stationary and non-stationary electromagnetic processes in the nonrelativistic and relativistic backward-wave tubes with maximal accounting for the different physical factors such as the relativistic effects, effects of dissipation, the presence of a space charge, wave reflections at the ends of the slowing system, stochastic factors by means including the special elements in a whole system etc as well as the detailed investigation of characteristics (dynamical and topological invariants) of dynamics of the backward-wave tube in regular, automodulation and chaotic regimes with construction the corresponding bi-furcation diagrams. Below in order to further solve the system of differential equations for the backward-wave tube it is in brief described the realizing numerical scheme, based on the use of the conservative finite-difference schemes of the "predictor-corrector" type and the method of the sweep method.

2. Standard system of differential equations for a backward-wave tube

In the ref. [1-8] it has presented the detailed explanation of the master systems of equations describing the fundamental processes in the system, so we should only present the corresponding system in the suitable form. According to ref. [3-6], the component of the wave field that interacts with the electron beam can be represented as

$$E(x,t)=R\left[\widetilde{E}(x,t)e^{i\omega_0 t-i\beta_0 x}\right],$$

where $\widetilde{E}(x,t)$ is a slowly varying complex function.

The high-frequency current arising in the electron beam as a result of the action of the wave field on it is represented in the following form [5]:

$$I(x,t)=Re\left[I_1(x,t)e^{i\omega_0\left(t-\frac{x}{v_0}\right)}+I_2(x,t)e^{2i\omega_0(t-x/v_0)}+\dots\right], \qquad (1)$$

where $I_1(x,t)$, $I_2(x,t)$... – slow amplitudes of the first, second and subsequent harmonics

First master equation can be written in the standard form (see details in Refs. [3-5]):

$$\frac{1}{\upsilon_{гр}}\frac{\partial\widetilde{E}}{\partial t}-\frac{\partial\widetilde{E}}{\partial x}=\frac{-1}{2}\beta_0^2K_0\widetilde{I}_1, \qquad (2)$$

where K_0 – the coupling resistance of the retarder system for the working spatial harmonic at the frequency ω_0.

The second component of the self-consistent theory is the formulation of the electron motion equation which has the standard form [7,8]:

$$\mathrm{w}\frac{d^2x}{dt^2}=\frac{e}{m_0}\mathrm{Re}\left[\tilde{E}(x,t)e^{i\omega_0 t-i\beta_0 x}\right] \qquad (3)$$

Further usually one could define $t(x,t_0)=t_0+x/\upsilon_0+\tilde{t}(x,t_0)$ as a time of arrival at the point x of the electron that flew into the interaction space at the moment t_0.

Due to the slowness of the change of the complex amplitude in time and the smallness of the change in the speed of electrons in the process of interaction, one has the right to replace in the right part $\widetilde{E}(x,t(x,t_0))$ на

$\widetilde{E}\left(x, t_0+x/\upsilon_0\right)$ i $\left(\partial t/\partial x\right)^3_{t_0}$ on υ_0^{-3}, and further it can be written as:

$$(\partial^2\tilde{t}/\partial x^2)_{t_0} = -\frac{e}{mv_0^3}Re\left[\tilde{E}(x, t + x/\upsilon_0)e^{i\omega_0(t_0+\tilde{t}(x,t_0))}\right]. \quad (4)$$

After reduction to dimensionless variables and parameters, the equations and boundary conditions take the following form [4-7]:

$$\partial^2\theta/\partial\zeta^2 = -R\left[Fexp(i\theta)\right],$$

$$\partial F/\partial\tau - \partial F/\partial\zeta = \tilde{I},$$

$$\tilde{I} = \frac{-1}{\pi}\int_0^{2\pi} e^{-i\theta} d\theta_0, \quad (5)$$

$$\theta\square_{\zeta=0} = \theta_0, \ \partial\theta/\partial\zeta\square_{\zeta=0} = 0,$$

$$F\square_{\zeta=L} = 0, \quad (6)$$

where $\zeta = \beta_0 Cx$,

$$\tau = \omega_0 C\left(t - x/\upsilon_0\right)\left(1+\upsilon_0/\upsilon_{гр}\right)^{-1} - \quad (7)$$

are the dimensionless independent variables are coordinate and "local time". Due to the introduction of "local time", which is counted at each point of the space of interaction with the displacement x/υ_0, there is no derivative in the equation of motion by τ which facilitates the construction of a difference scheme for the numerical solution of the system of equations. This is also convenient in the sense that in the dimensionless form of the equations, the parameter of the ratio of the group velocity to the beam velocity is excluded, which now appears only in the coefficient connecting dimensional and dimensionless time. As usually, the value $\theta(\zeta, \tau, \theta_0)$characterizes the phase relative to the wave for an electron that has flown into the space of interaction with the phase θ_0 , and

$$F(\zeta, \tau) = \widetilde{E}/\left(2\beta_0 UC^2\right)$$

is the dimensionless complex amplitude of the high-frequency wave field. Pierce's parameter

$$C = \sqrt[3]{I_0 K_0/(4U)},$$

where is I_0 the constant component of the beam current, U is the accelerating voltage, assumed to be small.

3. Numerical differences scheme

In order to further solve the system of differential equations for the backward-wave tube it has been developed a numerical scheme, which is based on the use of finite-difference schemes of the "predictor-corrector" type and the method of the sweep method (see details in refs. [3-7]). The finite-difference scheme has been constructed and analogous the scheme [5,6]. Namely, the following substitutions have been applied:

$$[Q = exp(i\theta)],$$

$$X = (1/L)d\theta)/d\zeta, \quad (8)$$

the corresponding master system:

$$\partial^2\theta/\partial\zeta^2 = -Re[F\,exp(i\theta)],$$

$$\partial F/\partial\tau - \partial F/\partial\zeta = \tilde{I},$$

$$\tilde{I} = -\frac{1}{\pi}\int_0^{2\pi} e^{-i\theta}\, d\theta_0, \quad (9)$$

has the following form:

$$\frac{\partial Q}{\partial} = iLXQ$$

$$\frac{\partial X}{\partial} = -LRe(FQ) \quad (10)$$

Further the initial filed distribution is determined in the points of the net on the coordinate. The known method of largest particles is used. As usually the initial distribution of a field is determined by the distribution of the main amplitude mode. For each particle (index j) and each time moment (index i) it is solving the equation of motion according to the standard predictor—corrector scheme:

$$X^j_{i+\frac{1}{2}} = X^j_i + LP^j_i\frac{d}{2} = X^j_i + DP^j_i \quad (11)$$

$$Q^j_{i+\frac{1}{2}} = Q^j_i + iL X^j_i Q^j_i \frac{d}{2} = Q^j_i + iDL X^j_i Q^j_i$$

where

$$D = L\frac{d\zeta}{2},$$

$$P = -Re[FQ]$$

is a particle acceleration

Then it has been calculating (at the next step):

$$P^j_{i+\frac{1}{2}} = -Re\{[(F_i + F_{i+1})/2]QX^j_{i+\frac{1}{2}}\},$$

$$X^j_{i+1} = X^j_i + LP^j_{i+1/2}d\zeta = X^j_i + 2DP^j_{i+1/2},$$

$$Q^j_{i+1} = Q^j_i + iLX^j_{i+1/2}Q^j_{i+1/2}d\zeta = Q^j_i + 2iDLX^j_{i+1/2}Q^j_{i+1/2}, \quad (12)$$

The values of the first harmonic of the current in each node are determined by the standard expression:

$$I_i = (2/J)\sum_{j=1}^{J} Q^j_i$$

After determining the current values at each point of the grid, the induced field is then determined. To solve the excitation equation, a second-order scheme is used; accordingly, the iterative procedure for calculating the next time value of the field at each node along the coordinate has the form:

$$F^{l+1}_i = a_1F^l_{i+1} + a_2F^l_i + a_3F^l_{i-1} - D[b_1I^l_{i+}b_2I^{l-1}_i + b_3I^l_{i-1}] \quad (13)$$

Other details can be found in the refs. [3-10]. In conclusions let us note that it is supposed the further generalization and development more advanced approach with maximal accounting for the different physical factors (such as the relativistic effects, effects of dissipation, the presence of a space charge, wave reflections at the ends of the slowing system, stochastic factors by means including the special elements in a whole system etc) as well as the further detailed numerical investigation of characteristics (dynamical and topological invariants) of dynamics of nonrelativistic and relativistic backward-wave tubes (carcinotrons) in different regular, automodulation and chaotic (hyperchaotic) regimes with construction the corresponding bi-furcation diagrams and understanding the features of so called (generally speaking) self-oscillating distributed dynamical systems, which are characterized by the absence of rigid scenarios of the transition to chaos, in contrast to common known scenarios in more simple systems.

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Summary. It is developed an effective computational approach to solution the master corresponding system of differential equations, which describe the nonlinear stationary and non-stationary electromagnetic processes in the nonrelativistic and relativistic backward-wave tubes (carcinotrons) with maximal accounting for the different physical factors such as the relativistic effects, effects of dissipation, the presence of a space charge, wave reflections at the ends of the slowing system, stochastic factors by means including the special elements in a whole system etc as well as the detailed investigation of characteristics (dynamical and topological invariants) of dynamics of a carcinotron in automodulation and chaotic regimes with construction the corresponding bi-furcation diagrams. Below in order to further solve the master system of dynamical equations for carcinotron it is presented in brief the realizing numerical scheme, based on the use of the conservative finite-difference schemes of the "predictor-corrector" type and the sweep algorithm.

Key words: nonrelativistic and relativistic backward-wave tubes, carcinotrons, computational approach, chaotic dynamics, difference scheme and sweep algorithm

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Ткач Т.Б., Квасикова А.С., Шпінарева І.М.

ПРО ДЕЯКУЮ ЧИСЛОВУ МОДЕЛЬ ДО РОЗВ'ЯЗУВАННЯ ДИНАМІЧНИХ РІВНЯНЬ НЕРЕЛЯТИВІСТСЬКОЇ ТА РЕЛЯТИВІСТСЬКОЇ ЛАМПИ ЗВОРОТНОЇ ХВИЛІ

Резюме. Розроблється ефективний обчислювальний підхід до розв'язання відповідної мастерної системи динамічних рівнянь, які описують нелінійні стаціонарні та нестаціонарні електромагнітні процеси в нерелятивістських та релятивістських лампах зворотної хвилі (карсинотронах) з подальшим максимальним урахуванням різних фізичних факторів, таких як релятивістські ефекти. , ефекти дисипації, наявність просторового заряду, відбиття хвилі на кінцях сповільнюваної системи, стохастичні фактори за допомогою включення спеціальних елементів у всю систему тощо, а також дослідженням характеристик (динамічних і топологічних інваріантів) динаміка карсинотрона в автомодуляційному та хаотичному режимах з побудовою відповідних біфуркаційних діаграм. Для розв'язування головної системи динамічних рівнянь карсинотрона наведено реалізуєму чисельну схему, яка заснована на використанні консервативної різницевої схеми типу «предиктор-коректор» та алгоритму розгортки.

Ключові слова: нерелятивістські та релятивістські лампи зворотної хвилі, карсинотрон, обчислювальний підхід, хаотична динаміка, різницева схема та алгоритм розгортки

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